College Prep Stats
7.2 ~ 7.3 Extra Practice

1: What proportion p of U.S. high school students smoke? The 2007 Youth Risk Behavioral Survey questioned a random sample of 14,041 students in grades 9 to 12. Of these, 2808 said they had smoked cigarettes at least one day in the past month.

$$\hat{p} = \frac{2808}{14041}$$

2: A Pew Research Center poll, 70% of 1501 randomly selected adults in the United States believe in global warming, so the sample proportion is $\hat{p}$ = 0.70. Find the best point estimate of the proportion of all adults in the United States who believe in global warming.

(1) Poll = SRS,
(2) n = 1501 fixed, independent, two outcomes, prob. same
(3) $n\hat{p} = 1501 * 0.7 = 1050.7, \quad n\hat{q} = 1501 * 0.3 = 450.3$

$$\hat{p} = 0.7 = 70\%$$

3: The 96% confidence interval for the true proportion of all 17 year old boys who own a used car with a sample size of 426 is (0.189, 0.251). Interpret this confidence interval.

We are 96% confident that the true population proportion of 17 year old boys who own a used car is between 0.189 to 0.251.

4: The 95% confidence interval for the true proportion of all New York State Union members who favor the Republican candidate for governor with a sample size of 300 is (0.189, 0.251). Interpret this confidence interval.

We are 95% confident that the true population proportion of all New York State Union members who favor the Republican candidate for governor is between 0.189 to 0.251.

5: Find the appropriate critical value for the given confidence level.

a) 99.9% b) 95% c) 90%

d) 92% e) 84% f) 78%

a) 3.2905 d) 1.7507 e) 1.4051 f) 1.2265

6: Assume that a sample is used to estimate a population proportion p. Find the margin of error, E, that corresponds to the given statistics and confidence level. Round the margin of error to four decimal places.

a) 95% confidence, sample size is 600, of 32% are successes.

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{0.32(0.68)}{600}} \approx 0.0373$$

b) 98% confidence, sample size is 1142, of 24% are successes.

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.3263\sqrt{\frac{0.24(0.76)}{1142}} \approx 0.0294$$

7: In a simple random sample of 203 college students, 75 had part-time jobs. Find the margin of error for the 90% confidence interval used to estimate the population proportion.

(1) SRS,
(2) n = 203 fixed, independent, two outcomes, prob. same
(3) $x = 75 > 5, \quad n - x = 203 - 75 = 128 > 5$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.6449\sqrt{\frac{75/203(128/203)}{203}} \approx 0.0557$$

3

8: A Pew Research Center poll of 1501 randomly selected U.S. adults showed that 70% of the respondents believe in global warming. The sample results are n = 1501 and $\hat{p}$ = 0.70.

a) Find the margin of error E that corresponds to a 95% confidence level.

(1) Poll = SRS,
(2) n = 1501 fixed, independent, two outcomes, prob. same
(3) $n\hat{p} = 1501 * 0.7 = 1050.7, \quad n\hat{q} = 1501 * 0.3 = 450.3$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{0.7(0.3)}{1501}} \approx 0.0232$$

b) Find the 95% confidence interval estimate of the population proportion p.

95% C.I. is (0.7 – 0.0232, 0.7 + 0.0232),
or, (0.6768, 0.7232)

9: Use the given degree of confidence and sample data to construct a confidence interval for the population proportion p.
n = sample size, x = number of successes

a) n = 741, x = 274; 95% confidence
$x = 274 > 5, \; n - x = 741 - 274 = 467 > 5, \; n = 741, \; \alpha = 0.05,$
$z_{\alpha/2} = z_{0.025} = 1.96$
$$E = z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{274/741*467/741}{741}} \approx 0.0348$$
95% C. I. is (0.3350, 0.4045)

b) n = 267, x = 194; 88% confidence
$x = 194 > 5, \; n - x = 267 - 194 = 73 > 5, \; n = 267, \; \alpha = 0.12,$
$z_{\alpha/2} = z_{0.06} = 1.5548$
$$E = z_{0.06}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.5548\sqrt{\frac{194/267*73/267}{267}} \approx 0.0424$$
88% C. I. is (0.6842, 0.7690)

10: Use the given data to find the minimum sample size required to estimate the population proportion.

a) Margin of error: 0.006; confidence level: 90%; $\hat{p}$ and $\hat{q}$ unknown.

$$n = \frac{(z_{\alpha/2})^2 0.25}{E^2} = \frac{1.6449^2 * 0.25}{0.006^2} \approx 18788.49 \approx 18789$$

b) Margin of error: 0.02; confidence level: 94%; $\hat{p}$ and $\hat{q}$ unknown.

$$n = \frac{(z_{\alpha/2})^2 0.25}{E^2} = \frac{1.8808^2 * 0.25}{0.02^2} \approx 2210.87 \approx 2211$$

11: Use the given data to find the minimum sample size required to estimate the population proportion.

a) Margin of error: 0.08; confidence level: 96%; from a prior study, $\hat{p}$ is estimated by 0.24.

$$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2} = \frac{2.0537^2 * 0.24 * 0.76}{0.08^2} \approx 120.21 \approx 121$$

b) Margin of error: 0.004; confidence level: 92%; from a prior study, $\hat{p}$ is estimated by 0.123.

$$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2} = \frac{1.7507^2 * 0.123 * 0.877}{0.004^2} \approx 20663.38 \approx 20664$$

12: The Genetics and IVF Institute conducted a clinical trial of the XSORT method designed to increase the probability of conceiving a girl. As of this writing, 574 babies were born to parents using the XSORT method, and 525 of them were girls.

a) What is the best point estimate of the population proportion of girls born to parents using the XSORT method?

(1) Trial = SRS,
(2) n = 574 fixed, independent, two outcomes, prob. same
(3) $x = 525 > 5, \quad n - x = 574 - 525 = 49 > 5$

$$\hat{p} = \frac{525}{574}$$

b) Use the sample data to construct a 95% confidence interval estimate of the percentage of girls born to parents using the XSORT method.

$$E = z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{525/574 * 49/574}{574}} \approx 0.0229$$

95% C. I. is (0.8918, 0.9375)

13: An interesting and popular hypothesis is that individuals can temporarily postpone their death to survive a major holiday or important event Such as a birthday. In a study of this phenomenon, it was found that in the week before and the week after Thanksgiving, there were 12,000 total deaths, and 6,062of them occurred in the week before Thanksgiving (based on data from "Holiday, Birthdays, and Postponement of Cancer Death," by Young and Hade, *Journal of the American Medical Association*, Vol. 292, No. 24.)

a) What is the best point estimate of the proportion of deaths in the week before Thanksgiving to the total deaths in the week before and the week after Thanksgiving?

(1) Study = SRS,
(2) n = 12000 fixed, independent, two outcomes, prob. same
(3) $x = 6062 > 5, \quad n - x = 12000 - 6062 = 5938 > 5$

$$\hat{p} = \frac{6062}{12000} = \frac{3031}{6000}$$

b) Construct a 95% confidence interval estimate of the proportion of deaths in the week before Thanksgiving to the total deaths in the week before and the week after Thanksgiving?

$$E = z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{3031/6000 * 2969/6000}{12000}} \approx 0.0089$$

95% C. I. is (0.4962, 0.5141)

14: The following confidence interval is obtained for a population proportion, p: (0.426, 0.612). Use these confidence interval limits to find the point estimate, $\hat{p}$.

$$\hat{p} = \frac{0.612 + 0.426}{2} = 0.519$$

15: The following confidence interval is obtained for a population proportion, p: $0.542 < p < 0.714$. Use these confidence interval limits to find the margin of error, E.

$$E = \frac{0.714 - 0.542}{2} = 0.086$$

16: Use the given confidence level and sample data to find the margin of error, E. College students' annual earnings: 99% confidence, $n = 74$, $\bar{x} = \$3967$, $\sigma = \$874$

$$E = z_{0.005}\frac{\sigma}{\sqrt{n}} = 2.5758\frac{874}{\sqrt{74}} \approx 261.71$$

17: Using the information above, construct a 99% confidence interval estimate of the mean annual earnings of college students.

95% C. I. is (3967 – 261.71 , 3967 + 261.71)
or, (3705.29, 4228.71)

18: Assume that we want to estimate the mean IQ score for the population of statistics students. How many statistics students must be randomly selected for IQ tests if we want 99% confidence that the sample mean is within 2 IQ points of the population mean? (Use $\sigma = 15$.)

CL = 99%, $\alpha = 0.01$, $E = 2$,

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.5758*15}{2}\right]^2 \approx 373.21 \approx 374$$

19: A sample of size n = 100 produced the sample mean of $\bar{x} = 16$. Assuming the population standard deviation $\sigma = 3$, compute a 95% confidence interval for the population mean μ.

$$E = z_{0.025}\frac{\sigma}{\sqrt{n}} = 1.96\frac{3}{\sqrt{100}} \approx 0.5880$$

95% C. I. is (16 – 0.5880, 16 + 0.5880), or, (15.412, 16.588)

20: Assuming the population standard deviation $\sigma = 3$, how large should a sample be to estimate the population mean μ with a margin of error not exceeding 0.5?

If the CL is not given, you can assume CL = 95% (the median common CL), $\alpha = 0.05$, $E = 0.5$,

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96*3}{0.5}\right]^2 \approx 138.29 \approx 139$$

21: The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes.

a) After observing 120 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a 92% confidence interval for the mean assembly time.

(1) In a large production plant, we can assume that there exist an Quality Control Department. The QC engineer there knows how to take a SRS.
(2) $\sigma = 3.6$ is known
(3) $n = 120 > 30$

$\bar{x} = 16.2$

$$E = z_{0.04}\frac{\sigma}{\sqrt{n}} = 1.7507 * \frac{3.6}{\sqrt{120}} \approx 0.5753$$

92% C. I. is (15.6247, 16.7753)

b) How many workers should be involved in this study in order to have the mean assembly time estimated up to ±15 seconds with 92% cconfidence.

CL = 92%, $\alpha = 0.08$, $E = 0.25$ (15 sec = 15/60 = 0.25 min),

$z_{\alpha/2} = z_{0.04} = 1.7507$

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.7507*3.6}{0.25}\right]^2 \approx 635.54 \approx 636$$