1-3 Continuity, End Behavior, and Limits

Determine whether each function is continuous at the given *x*-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite,jump* , or *removable*.

1. $f(x) = \sqrt{x^2 - 4}$; at $x = -5$

ANSWER:

Continuous; $f(-5) = \sqrt{21} \approx 4.58$, $\lim_{x \to -5} f(x) \approx 4.58$, and $\lim_{x \to -5} f(x) = f(-5)$.

2. $f(x) = \sqrt{x + 5}$; at $x = 8$

ANSWER:

Continuous; $f(8) = \sqrt{13}$ or about 3.606, $\lim_{x \to 8} f(x) \approx 3.606$, and $\lim_{x \to 8} f(x) = f(8)$.

3. $h(x) = \dfrac{x^2 - 36}{x + 6}$; at $x = -6$ and $x = 6$

ANSWER:

Discontinuous at $x = -6$; $h(-6)$ is undefined and $\lim_{x \to -6} h(x) = -12$, so $h(x)$ has a removable discontinuity at $x = -6$.

Continuous at $x = 6$. $h(6) = 0$, $\lim_{x \to 6} h(x) = 0$, and $\lim_{x \to 6} h(x) = h(6)$.

4. $h(x) = \dfrac{x^2 - 25}{x + 5}$; at $x = -5$ and $x = 5$

ANSWER:

Discontinuous at $x = -5$; $h(-5)$ is undefined and $\lim_{x \to -5} h(x) = -10$, so $h(x)$ has a removable discontinuity at $x = -5$.

Continuous at $x = 5$. $h(5) = 0$, $\lim_{x \to 5} h(x) = 0$, and $\lim_{x \to 5} h(x) = h(5)$.

5. $g(x) = \dfrac{x}{x - 1}$; at $x = 1$

ANSWER:

Discontinuous; $g(1)$ is undefined and $g(x)$ approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right, so $g(x)$ has an infinite discontinuity at $x = 1$.

6. $g(x) = \dfrac{2 - x}{2 + x}$; at $x = 2$ and $x = -2$

ANSWER:

Discontinuous at $x = -2$; $g(-2)$ is undefined and $g(x)$ approaches $-\infty$ as x approaches -2 from the left and ∞ as x approaches -2 from the right, so $g(x)$ has an infinite discontinuity at $x = -2$. Continuous at $x = 2$; $g(2) = 0$, $\lim_{x \to 2} g(x) = 0$, and $\lim_{x \to 2} g(x) = g(2)$.

7. $h(x) = \dfrac{x-4}{x^2-5x+4}$; at $x = 1$ and $x = 4$

ANSWER:

Discontinuous at $x = 1$; $h(1)$ is undefined and $h(x)$ approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right, so $h(x)$ has an infinite discontinuity at $x = 1$. Discontinuous at $x = 4$; $h(4)$ is undefined and $\lim_{x \to 4} h(x) = \frac{1}{3}$, so $h(x)$ has a removable discontinuity at $x = 4$.

8. $h(x) = \dfrac{x(x-6)}{x^3}$; at $x = 0$ and $x = 6$

ANSWER:

Discontinuous at $x = 0$; $h(0)$ is undefined and $h(x)$ approaches $-\infty$ as x approaches 0 from both sides, so $h(x)$ has an infinite discontinuity at $x = 0$. Continuous at $x = 6$; $h(6) = 0$, $\lim_{x \to 6} h(x) = 0$, and $\lim_{x \to 6} h(x) = h(6)$.

9. $f(x) = \begin{cases} 4x-1 & \text{if } x \le -6 \\ -x+2 & \text{if } x > -6 \end{cases}$; at $x = -6$

ANSWER:

Discontinuous at $x = -6$; $f(x)$ approaches –25 as x approaches –6 from the left and 8 as x approaches –6 from the right, so $f(x)$ has a jump discontinuity at $x = -6$.

10. $f(x) = \begin{cases} x^2-1 & \text{if } x > -2 \\ x-5 & \text{if } x \le -2 \end{cases}$; at $x = -2$

ANSWER:

Discontinuous at $x = -2$; $f(x)$ approaches –7 as x approaches –2 from the left and 3 as x approaches –2 from the right, so $f(x)$ has a jump discontinuity at $x = -2$.