1-3 Continuity, End Behavior, and Limits

Determine whether each function is continuous at the given *x*-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*,*jump* , or *removable*.

2. $f(x) = \sqrt{x+5}$; at $x = 8$

ANSWER:

Continuous; $f(8) = \sqrt{13}$ or about 3.606, $\lim_{x \to 8} f(x) \approx 3.606$, and $\lim_{x \to 8} f(x) = f(8)$.

4. $h(x) = \frac{x^2 - 25}{x + 5}$; at $x = -5$ and $x = 5$

ANSWER:

Discontinuous at $x = -5$; $h(-5)$ is undefined and $\lim_{x \to -5} h(x) = -10$, so $h(x)$ has a removable discontinuity at $x = -5$. Continuous at $x = 5$. $h(5) = 0$, $\lim_{x \to 5} h(x) = 0$, and $\lim_{x \to 5} h(x) = h(5)$.

6. $g(x) = \frac{2-x}{2+x}$; at $x = 2$ and $x = -2$

ANSWER:

Discontinuous at $x = -2$; $g(-2)$ is undefined and $g(x)$ approaches $-\infty$ as x approaches -2 from the left and ∞ as x approaches -2 from the right, so $g(x)$ has an infinite discontinuity at $x = -2$. Continuous at $x = 2$; $g(2) = 0$, $\lim_{x \to 2} g(x) = 0$, and $\lim_{x \to 2} g(x) = g(2)$.

8. $h(x) = \frac{x(x-6)}{x^3}$; at $x = 0$ and $x = 6$

ANSWER:

Discontinuous at $x = 0$; $h(0)$ is undefined and $h(x)$ approaches $-\infty$ as x approaches 0 from both sides, so $h(x)$ has an infinite discontinuity at $x = 0$. Continuous at $x = 6$; $h(6) = 0$, $\lim_{x \to 6} h(x) = 0$, and $\lim_{x \to 6} h(x) = h(6)$.

1-3 Continuity, End Behavior, and Limits

10. $f(x)=\begin{cases} x^2-1 & \text{if } x>-2 \\ x-5 & \text{if } x\le -2 \end{cases}$; at $x = -2$

ANSWER:

Discontinuous at $x = -2$; $f(x)$ approaches -7 as x approaches -2 from the left and 3 as x approaches -2 from the right, so $f(x)$ has a jump discontinuity at $x = -2$.

Determine between which consecutive integers the real zeros of each function are located on the given interval.

14. $g(x) = -x^3 + 6x + 2$; [−4, 4]

ANSWER:

−3 and −2, −1 and 0, 2 and 3

16. $h(x) = -x^4 + 4x^3 - 5x - 6$; [3, 5]

ANSWER:

3 and 4

18. $g(x) = \dfrac{x^2 + 3x - 3}{x^2 + 1}$; [−4, 3]

ANSWER:

−4 and −3, 0 and 1

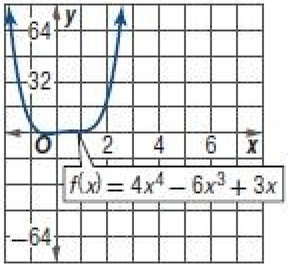
20. $f(x) = \sqrt{x^2 - 6}$; [3, 8]

ANSWER:

6 and 7

1-3 Continuity, End Behavior, and Limits

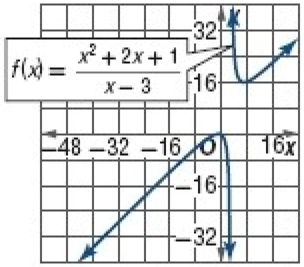
Use the graph of each function to describe its end behavior. Support the conjecture numerically.

22. 

ANSWER:

From the graph, it appears that $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

x	f(x)
–10,000	$4 \cdot 10^{16}$
–1000	$4 \cdot 10^{12}$
0	0
1000	$4 \cdot 10^{12}$
10,000	$4 \cdot 10^{16}$

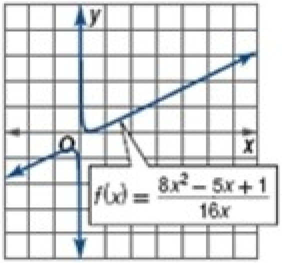


24.

ANSWER:

From the graph, it appears that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

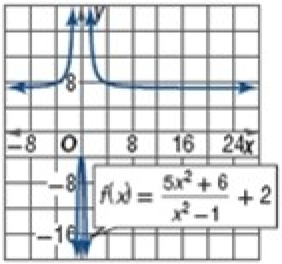
x	f(x)
–10,000	–9995
–1000	–995
0	–0.3333
1000	1005
10,000	10,005

26. 

ANSWER:

From the graph, it appears that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

x	f(x)
–10,000	–5000
–1000	–500
0	undefined
1000	499.7
10,000	4999.7

28. 

ANSWER:

From the graph, it appears that $f(x) \to 7$ as $x \to -\infty$ and $f(x) \to 7$ as $x \to \infty$.

x	f(x)
–10,000	7.0000001
–1000	7.00001
0	–4
1000	7.00001
10,000	7.0000001