1-6 Function Operations and Composition of Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

2. $f(x) = 8 - x^3$
$g(x) = x - 3$

ANSWER:

$(f + g)(x) = -x^3 + x + 5$; D = $(-\infty, \infty)$; $(f - g)(x) = -x^3 - x + 11$; D = $(-\infty, \infty)$; $(f \cdot g)(x) = -x^4 + 3x^3 + 8x - 24$; D = $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{8 - x^3}{x - 3}$;
$D = (-\infty, 3) \cup (3, \infty)$

4. $f(x) = x - 9$
$g(x) = x + 5$

ANSWER:

$(f + g)(x) = 2x - 4$; D = $(-\infty, \infty)$; $(f - g)(x) = -14$; D = $(-\infty, \infty)$; $(f \cdot g)(x) = x^2 - 4x - 45$; D = $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x - 9}{x + 5}$; $D = (-\infty, -5) \cup (-5, \infty)$

6. $f(x) = x - 7$
$g(x) = x + 7$

ANSWER:

$(f + g)(x) = 2x$; D = $(-\infty, \infty)$; $(f - g)(x) = -14$; D = $(-\infty, \infty)$; $(f \cdot g)(x) = x^2 - 49$; D = $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x - 7}{x + 7}$; $D = (-\infty, -7) \cup (-7, \infty)$

8. $f(x) = \frac{x}{4}$

$g(x) = \frac{3}{x}$

ANSWER:

$(f + g)(x) = \frac{x^2 + 12}{4x}$; $D = (-\infty, 0) \cup (0, \infty)$; $(f - g)(x) = \frac{x^2 - 12}{4x}$; $D = (-\infty, 0) \cup (0, \infty)$; $(f \cdot g)(x) = \frac{3}{4}$; $D = (-\infty, 0) \cup (0, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x^2}{12}$;
$D = (-\infty, 0) \cup (0, \infty)$

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10. $f(x) = \frac{3}{x}$

$g(x) = x^4$

ANSWER:

$(f + g)(x) = \frac{3}{x} + x^4$; D = $(-\infty, 0) \cup (0, \infty)$; $(f - g)(x) = \frac{3}{x} - x^4$; D = $(-\infty, 0) \cup (0, \infty)$; $(f \cdot g)(x) = 3x^3$; D = $(-\infty, 0) \cup (0, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3}{x^5}$; D = $(-\infty, 0) \cup (0, \infty)$

12. $f(x) = \sqrt{x+6}$

$g(x) = \sqrt{x-4}$

ANSWER:

$(f + g)(x) = \sqrt{x+6} + \sqrt{x-4}$; D = [4, ∞); $(f - g)(x) = \sqrt{x+6} - \sqrt{x-4}$; D = [4, ∞); $(f \cdot g)(x) = \sqrt{x^2 + 2x - 24}$; D = [4, ∞); $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x^2 + 2x - 24}}{x-4}$; D = (4, ∞)

For each pair of functions, find [f o g](x), [g o f](x), and [f o g](6).

16. $f(x) = -2x^2 - 5x + 1$

$g(x) = -5x + 6$

ANSWER:

[f o g](x) = $-50x^2 + 145x - 101$; [g o f](x) = $10x^2 + 25x + 1$; [f o g](6) = -1031

18. $f(x) = x^2 - 16$

$g(x) = x^2 + 7x + 11$

ANSWER:

[f o g](x) = $x^4 + 14x^3 + 71x^2 + 154x + 105$; [$g$ o f](x) = $x^4 - 25x^2 + 155$; [f o g](6) = 7905

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20. $f(x) = 2 + x^4$

$g(x) = -x^2$

ANSWER:

$[f \circ g](x) = 2 + x^8$; $[g \circ f](x) = -x^8 - 4x^4 - 4$; $[f \circ g](6) = 1{,}679{,}618$

Find $f \circ g$.

22. $f(x) = \dfrac{2}{x-3}$

$g(x) = x^2 + 6$

ANSWER:

$[f \circ g](x) = \dfrac{2}{x^2+3}$

24. $f(x) = x^2 - 9$

$g(x) = \sqrt{x+3}$

ANSWER:

$[f \circ g](x) = x - 6$ for $x \geq -3$

26. $f(x) = -\dfrac{4}{x}$

$g(x) = \sqrt{x+8}$

ANSWER:

$[f \circ g](x) = \dfrac{-4\sqrt{x+8}}{x+8}$ for $x > -8$

28. $f(x) = \sqrt{x-2}$

$g(x) = x^2 + 8$

ANSWER:

$[f \circ g](x) = \sqrt{x^2 + 6}$