1-7 Inverse Relations and Functions

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

4. $f(x) = 3x - 8$

ANSWER:

yes

8. $f(x) = -4x^2 + 8$

ANSWER:

no

12. $f(x) = \frac{1}{4}x^3$

ANSWER:

yes

Determine whether each function has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

16. $f(x) = \sqrt{x+8}$

ANSWER:

yes; $f^{-1}(x) = x^2 - 8, x \geq 0$

20. $g(x) = \frac{x-6}{x}$

ANSWER:

yes; $g^{-1}(x) = \frac{-6}{x-1}, x \neq 1$

24. $h(x) = \dfrac{x+4}{3x-5}$

ANSWER:

yes; $h^{-1}(x) = \dfrac{5x+4}{3x-1}, x \neq \dfrac{1}{3}$

Show algebraically that f and g are inverse functions.

28. $f(x) = 4x + 9$

$g(x) = \dfrac{x-9}{4}$

ANSWER:

$$\begin{aligned} f[g(x)] &= 4\left(\frac{x-9}{4}\right) + 9 \\ &= x - 9 + 9 \\ &= x \end{aligned}$$

$$\begin{aligned} g[f(x)] &= \frac{4x+9-9}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

32. $f(x) = (x+8)^{\frac{3}{2}}$

$g(x) = x^{\frac{2}{3}} - 8;\ x \geq 0$

ANSWER:

$$f[g(x)] = \left(x^{\frac{2}{3}} - 8 + 8\right)^{\frac{3}{2}}$$
$$= \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}}$$
$$= x$$
$$g[f(x)] = \left[(x+8)^{\frac{3}{2}}\right]^{\frac{2}{3}} - 8$$
$$= x + 8 - 8$$
$$= x$$

36. $f(x) = \dfrac{x-6}{x+2}$

$g(x) = \dfrac{2x+6}{1-x}$

ANSWER:

$$f[g(x)] = \frac{\frac{2x+6}{1-x} - 6}{\frac{2x+6}{1-x} + 2}$$

$$= \frac{\frac{2x+6}{1-x} - \frac{6(1-x)}{1-x}}{\frac{2x+6}{1-x} + \frac{2(1-x)}{1-x}}$$

$$= \frac{\frac{2x+6-6+6x}{1-x}}{\frac{2x+6+2-2x}{1-x}}$$

$$= \frac{2x+6-6+6x}{2x+6+2-2x}$$

$$= \frac{8x}{8}$$

$$= x$$

$$g[f(x)] = \frac{2\left(\frac{x-6}{x+2}\right) + 6}{1 - \frac{x-6}{x+2}}$$

$$= \frac{\frac{2(x-6)}{x+2} + \frac{6(x+2)}{x+2}}{\frac{x+2}{x+2} - \frac{x-6}{x+2}}$$

$$= \frac{\frac{2x-12+6(x+2)}{x+2}}{\frac{x+2-(x-6)}{x+2}}$$

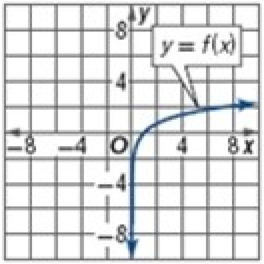
$$= \frac{2x-12+6x+12}{x+2-x+6}$$

$$= \frac{8x}{8}$$

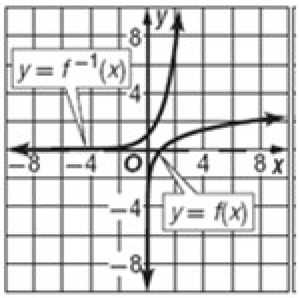
$$= x$$

1-7 Inverse Relations and Functions

Use the graph of each function to graph its inverse function.

40. 

ANSWER:



44. **JOBS** Jamie sells shoes at a department store after school. Her base salary each week is \$140, and she earns a 10% commission on each pair of shoes that she sells. Her total earnings $f(x)$ for a week in which she sold x dollars worth of shoes is $f(x) = 140 + 0.1x$.

a. Explain why the inverse function $f^{-1}(x)$ exists. Then find $f^{-1}(x)$.

b. What do $f^{-1}(x)$ and x represent in the inverse function?

c. What restrictions, if any, should be placed on the domains of $f(x)$ and $f^{-1}(x)$? Explain.

d. Find Jamie's total sales last week if her earnings for that week were \$220.

ANSWER:

a. Sample answer: The graph of the function is linear, so it passes the horizontal line test. Therefore, it is a one-to-one function and it has an inverse; $f^{-1}(x) = 10x - 1400$.

b. x represents Jamie's earnings for a week, and $f^{-1}(x)$ represents her sales.

c. $x \geq 0$, Jamie cannot have negative sales.

d. \$800