

$$a_n = a_1(r)^{n-1} \quad S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_n = \frac{a_1}{1-r}$$

1) In a geometric sequence $a_4 = -36$ and $a_7 = 972$. Find S_{19} .

$$\begin{aligned} a_4 &= a_1(r)^{4-1} \\ -36 &= a_1(r)^3 \\ a_7 &= a_1(r)^{7-1} \\ 972 &= a_1(r)^6 \end{aligned}$$

$$972 = \left(\frac{-36}{r^3} \right) r^6$$

$$-27 = r^3$$

$$r = -3 \quad \text{which means}$$

$$a_1 = \frac{4}{3}$$

$$S_{19} = \frac{4}{3} \left(\frac{1-(-3)^{19}}{1-(-3)} \right)$$

$$= 387,420,489.3$$

2) In a geometric sequence, $a_3 = -80$ and $a_8 = 2560$. Find S_{10} .

$$\begin{aligned} -80 &= a_1(r)^{3-1} \\ 2560 &= a_1(r)^{8-1} \\ -32 &= r^5 \\ r &= -2 \\ a_1 &= -20 \\ S_{10} &= 6820 \end{aligned}$$

For 3 – 5, find the sum of each infinite geometric series if possible.

3) $18 - 27 + 40.5 \dots$

$$\begin{aligned} a_1 &= 18 \\ r &= -1.5 \end{aligned} \quad \text{not possible}$$

4) $12 - 7.2 + 4.32 \dots$

$$\begin{aligned} r &= -\frac{3}{5} \\ S_n &= \frac{12}{1-(-\frac{3}{5})} \end{aligned}$$

5) $\sum_{n=1}^{\infty} 6(-0.4)^{n-1}$

$$\begin{aligned} a_1 &= 6(-.4)^0 \\ S_n &= \frac{a_1}{1-(-.4)} = \frac{6}{1.4} = \left(\frac{30}{7} \right) \end{aligned}$$

$$a_1 = 6$$

6) If $S_n = 1,007,769$ for the series $3 + 18 + 108 + 648 \dots$, find the value of n .

$$1,007,769 = 3 \left(\frac{1-(6)^n}{1-6} \right)$$

$$-1,679,615 = 1 - 6^n$$

$$1,679,616 = 6^n$$

$$\begin{aligned} \log_6 1,679,616 &= n \\ n &= 8 \end{aligned}$$

7) Given the series $\sum_{n=1}^{\infty} 5(2)^{n-1}$, find the value of n if $S_n = 315$.

$$315 = 5 \left(\frac{1-2^n}{1-2} \right)$$

$$-63 = 1 - 2^n$$

$$\log_2 64 = n$$

$$\begin{aligned} \log_2 2^6 &= n \\ 6 &= n \end{aligned}$$