12-2 Evaluating Limits Algebraically

Use the properties of limits to evaluate each limit.

2. $\lim\limits_{x \to 5} \dfrac{x^2 + 4x + 13}{x - 3}$

ANSWER:

29

4. $\lim\limits_{x \to -2} \dfrac{2x^5 - 4x^3 - 2x - 12}{x^3 + 5x^2}$

ANSWER:

$-\dfrac{10}{3}$

6. $\lim\limits_{x \to -4} \left[x^2(x+1) + 2 \right]$

ANSWER:

-46

8. $\lim\limits_{x \to 1} \dfrac{x^3 + 2x - 11}{x + 3}$

ANSWER:

-2

10. $\lim\limits_{x \to -6} \dfrac{x^4 - x^3}{x^2}$

ANSWER:

42

Use direct substitution, if possible, to evaluate each limit. If not possible, explain why not.

12. $\lim\limits_{x \to 2} \left(4x^3 - 3x^2 + 10 \right)$

ANSWER:

30

14. $\lim\limits_{x \to 3} \sqrt{2 - x}$

ANSWER:

Not possible; when $x = 3$, the function $f(x) = \sqrt{2 - x}$ is $\sqrt{-1}$, which is not a real number.

16. $\lim\limits_{x \to 4} \dfrac{x + 4}{x - 4}$

ANSWER:

Not possible; when $x = 4$, the denominator is 0.

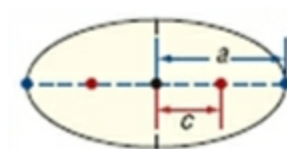18. $\lim\limits_{x \to 9} \left(3x^2 - 10x + 35 \right)$

ANSWER:

188

20. $\lim\limits_{x \to 1} \left(-x^2 + 3x + \sqrt{x} \right)$

ANSWER:

3

22. **GEOMETRY** The area of an ellipse is defined as $A = \pi a \sqrt{a^2 - c^2}$, where a is the distance from the vertices to the center and c is the distance from the foci to the center.

a. What is the area of an ellipse for $a = 5$ and $c = 3$?
b. What happens to the eccentricity of an ellipse as the foci move closer to the center of the ellipse?
c. What is the limit of the area of the ellipse as c approaches 0 in terms of a?

ANSWER:

a. 20π or 62.83 units^2
b. The eccentricity of the ellipse approaches 0, thus making the ellipse look more like a circle.
c. πa^2

Evaluate each limit.

24. $\lim\limits_{x \to 0} \dfrac{4x}{\sqrt{x+1} - 1}$

ANSWER:

8

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26. $\lim\limits_{x \to 9} \dfrac{\sqrt{x}-3}{x-9}$

    ANSWER:

    $\dfrac{1}{6}$

28. $\lim\limits_{x \to 7} \dfrac{5-\sqrt{18+x}}{x-7}$

    ANSWER:

    $-\dfrac{1}{10}$

30. $\lim\limits_{x \to \frac{1}{2}} \dfrac{8x^2+2x-3}{12x^2+8x-7}$

    ANSWER:

    $\dfrac{1}{2}$

32. $\lim\limits_{x \to -3} \dfrac{x^2-2x-15}{x+3}$

    ANSWER:

    -8

34. $\lim\limits_{x \to 0} \dfrac{\sqrt{16+x}-4}{x}$

    ANSWER:

    $\dfrac{1}{8}$

Evaluate each limit.

36. $\lim\limits_{x \to \infty} \dfrac{3x^3-10x+2}{4x^3+20x^2}$

    ANSWER:

    $\dfrac{3}{4}$

38. $\lim\limits_{x \to \infty} \left(10x+14+6x^2-x^4\right)$

    ANSWER:

    $-\infty$

40. $\lim\limits_{x \to \infty} \dfrac{14x^3-12x}{4x^2+13x-8}$

    ANSWER:

    ∞

42. $\lim\limits_{x \to \infty} \dfrac{6x^5-12x^2+14x}{2x^5+13x^3}$

    ANSWER:

    3

44. $\lim\limits_{x \to \infty} \dfrac{6x^3+2x-11}{-x^5+17x^3+4x}$

    ANSWER:

    0

46. $\lim\limits_{x \to -\infty} \left(2x^5-4x^2+10x-8\right)$

    ANSWER:

    $-\infty$