Name: Key

Area Under a Curve and Integration

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \qquad \Delta x = \frac{b-a}{n} \qquad x_i = a + i\Delta x$$

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1) Evaluate $\int_{-2}^{1} 2x\,dx$

$\Delta x = \frac{1-(-2)}{n} = \frac{3}{n}$

$x_i = -2 + i\left(\frac{3}{n}\right)$

$= -2 + \frac{3i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} 2(x_i)\left(\frac{3}{n}\right)$

$\lim \sum 2\left[-2+\frac{3i}{n}\right]\left(\frac{3}{n}\right)$

$\lim \sum \frac{-12}{n} + \frac{18i}{n^2}$

$\lim \frac{1}{n}\sum -12 + \frac{18}{n^2}\sum i$

$\lim \frac{1}{n}(-12n) + \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right)$

$\lim -12 + \frac{18n+18}{2n}$

$\lim -12 + 9 + \frac{9}{n}$

$-12 + 9 + 0$

(-3)

2) Evaluate $\int_{1}^{2} (x^2+1)\,dx$

$\Delta x = \frac{1}{n}$ $\quad x_i = 1 + i\left(\frac{1}{n}\right)$

$= 1 + \frac{i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[(x_i)^2 + 1\right]\left(\frac{1}{n}\right)$

$\lim \sum \left[\left(1+\frac{i}{n}\right)^2 + 1\right]\left(\frac{1}{n}\right)$

$\lim \sum \left(\left[1 + \frac{2i}{n} + \frac{i^2}{n^2}\right] + 1\right)\left(\frac{1}{n}\right)$

$\lim \sum \frac{1}{n} + \frac{2i}{n^2} + \frac{i^2}{n^3} + \frac{1}{n}$

$\lim \frac{1}{n}\sum 1 + \frac{2}{n^2}\sum i + \frac{1}{n^3}\sum i^2 + \frac{1}{n}\sum 1$

$\lim \frac{1}{n}(n) + \frac{2}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1}{n}(n$

$\lim 1 + \frac{2n+2}{2n} + \frac{2n^2+3n+1}{6n^2} + 1$

$1 + 1 + 0 + \frac{1}{3} + 0 + 0 + 1$

$3\frac{1}{3}$ or $\left(\frac{10}{3}\right)$