2-5 Rational Functions

Find the domain of each function and the equations of the vertical or horizontal asymptotes, if any.

2. $h(x) = \dfrac{x^3 - 8}{x + 4}$

ANSWER:

D = $\{x \mid x \neq -4, x \in \mathbb{R}\}$; $x = -4$

4. $g(x) = \dfrac{x - 6}{(x + 3)(x + 5)}$

ANSWER:

D = $\{x \mid x \neq -3, -5, x \in \mathbb{R}\}$; $x = -3$, $x = -5$, $y = 0$

6. $f(x) = \dfrac{x^2 + 9x + 20}{x - 4}$

ANSWER:

D = $\{x \mid x \neq 4, x \in \mathbb{R}\}$; $x = 4$

8. $g(x) = \dfrac{(x - 4)(x + 2)}{(x + 1)(x - 3)}$

ANSWER:

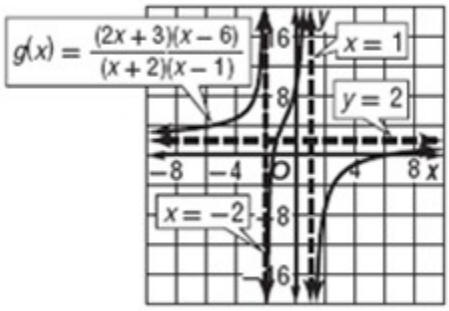
D = $\{x \mid x \neq 3, -1, x \in \mathbb{R}\}$; $x = 3$, $x = -1$, $y = 1$

For each function, determine any asymptotes and intercepts. Then graph the function and state its domain.

10. $g(x) = \dfrac{(2x + 3)(x - 6)}{(x + 2)(x - 1)}$

ANSWER:

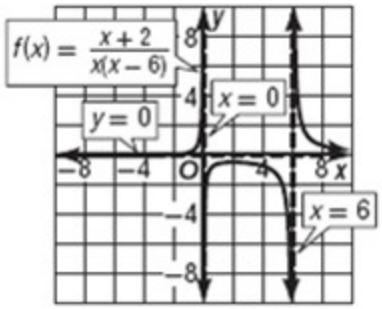
asymptotes: $x = 1$, $x = -2$, $y = 2$; x-intercepts: $-\dfrac{3}{2}, 6$; y-intercept: 9; D = $\{x \mid x \neq 1, -2, x \in \mathbb{R}\}$



12. $f(x) = \dfrac{x+2}{x(x-6)}$

ANSWER:

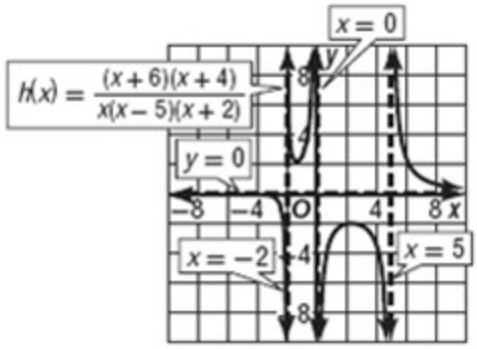
asymptotes: $x = 0$, $x = 6$, $y = 0$; x-intercept: -2; D = $\{x \mid x \neq 0, 6, x \in \mathbb{R}\}$



14. $h(x) = \dfrac{(x+6)(x+4)}{x(x-5)(x+2)}$

ANSWER:

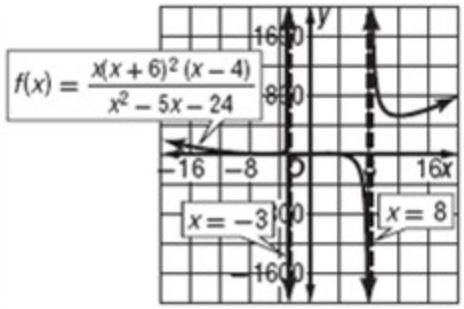
asymptotes: $x = 0$, $x = 5$, $x = -2$, $y = 0$; x-intercepts: -6, -4; D = $\{x \mid x \neq -2, 0, 5, x \in \mathbb{R}\}$



16. $f(x) = \dfrac{x(x+6)^2(x-4)}{x^2-5x-24}$

ANSWER:

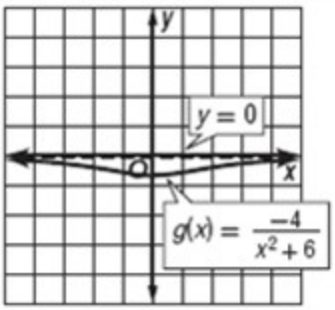
asymptotes: $x = -3$, $x = 8$; x-intercepts: -6, 0, 4; y-intercept: 0; D = $\{x \mid x \neq -3, 8, x \in \mathbb{R}\}$



18. $g(x) = \dfrac{-4}{x^2 + 6}$

ANSWER:

asymptote: $y = 0$; y-intercept: $-\dfrac{2}{3}$; D = $\{x \mid x \in \mathbb{R}\}$

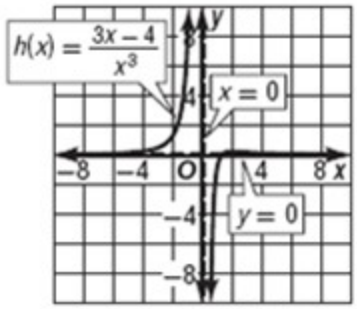


For each function, determine any asymptotes, holes, and intercepts. Then graph the function and state its domain.

20. $h(x) = \dfrac{3x - 4}{x^3}$

ANSWER:

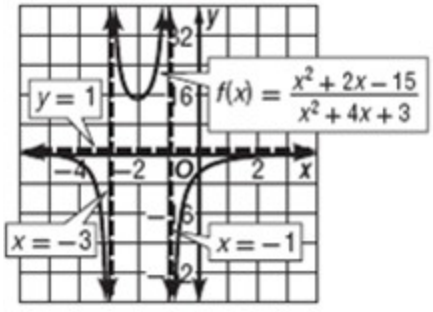
asymptotes: $x = 0, y = 0$; x-intercept: $\dfrac{4}{3}$; D = $\{x \mid x \neq 0, x \in \mathbb{R}\}$



22. $f(x) = \dfrac{x^2 + 2x - 15}{x^2 + 4x + 3}$

ANSWER:

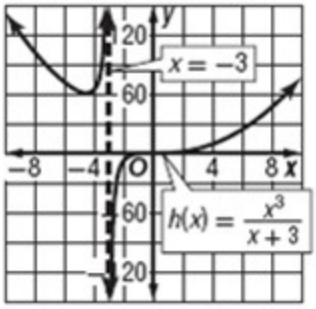
asymptotes: $y = 1$, $x = -3$, $x = -1$; x-intercepts: 3, –5; y-intercept: –5; D = $\{x \mid x \neq -3, -1, x \in \mathbb{R}\}$



24. $h(x) = \dfrac{x^3}{x+3}$

ANSWER:

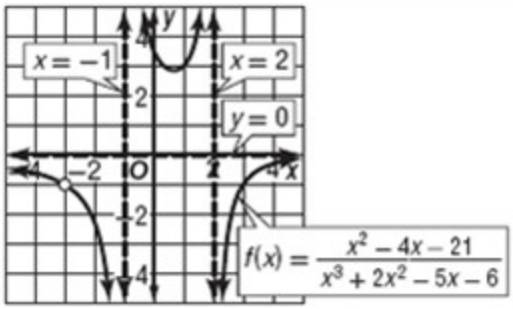
asymptote: $x = -3$; x-intercept: 0; y-intercept: 0; D = $\{x \mid x \neq -3, x \in \mathbb{R}\}$



26. $f(x) = \dfrac{x^2 - 4x - 21}{x^3 + 2x^2 - 5x - 6}$

ANSWER:

asymptotes: $x = -1$, $x = 2$, $y = 0$; hole: $(-3, -1)$; x-intercept: 7; y-intercept: $\dfrac{7}{2}$; D = $\{x \mid x \neq -3, -1, 2, x \in \mathbb{R}\}$



28. $f(x) = \dfrac{(x+4)(x-1)}{(x-1)(x+3)}$

ANSWER:

asymptotes: $x = -3, y = 1$; hole: $\left(1, \dfrac{5}{4}\right)$; x-intercept: -4; y-intercept: $\dfrac{4}{3}$; D = $\{x \mid x \neq -3, 1, x \in \mathbb{R}\}$