3-5 Modeling with Nonlinear Regression

For Exercises 1–3, complete each step.
a. Find an exponential function to model the data.
b. Find the value of each model at $x = 20$.

1.

x	y
1	7
2	11
3	25
4	47
5	96
6	193
7	380

ANSWER:

a. $y = 3.19(1.98)^x$
b. 2,605,304.4

2.

x	y
0	1
1	6
2	23
3	124
4	620
5	3130
6	15,600

ANSWER:

a. $y = 1.04(4.95)^x$
b. 8.09×10^{13}

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3.

x	y
0	25
1	6
2	1.6
3	0.4
4	0.09
5	0.023
6	0.006

ANSWER:

a. $y = 24.98(0.25)^x$
b. 1.98×10^{-11}

4. **GENETICS** *Drosophila melanogaster*, a species of fruit fly, are a common specimen in genetics labs because they reproduce about every 8.5 days, allowing researchers to study several generations. The table shows the population of *drosophila* over a period of days.

Generation	Drosophila	Generation	Drosophila
1	80	5	1180
2	156	6	2314
3	307	7	4512
4	593	8	8843

a. Find an exponential function to model the data.
b. Use the function to predict the population of drosophila after 93.5 days.

ANSWER:

a. $y = 40.69 \cdot 1.96^x$
b. 66,563

5. **SHARKS** Sharks have numerous rows of teeth embedded directly into their gums and not connected to their jaws. As a shark loses its teeth, teeth from the next row move forward. The rate of replacement of a row of teeth in days per row increases with the temperature of the water.

Temp. (°C)	20	21	22	23	24	25	26	27
Days per Row	66	54	44	35	28	22	18	16

a. Find an exponential function to model the data.
b. Use the function to predict the temperature at which sharks lose a row of teeth in 12 days.

ANSWER:

a. $y = 4476 \cdot 0.81^x$
b. 28.1°C

6. **WORDS** A word family consists of a base word and all of its derivations. The table shows the percentage of words in an average English text comprised of the most common word families.

Word Families	1000	2000	3000	4000	5000
Percentage of Words	73.1	79.7	84.0	86.7	88.6

a. Find a logarithmic function to model the data.
b. Predict the number of word families that make up 95% of the words in an average English text.

ANSWER:

a. $y = 5.84 + 9.74 \ln x$
b. 9483 word families

For Exercises 7–9, complete each step.
a. Find a logarithmic function to model the data.
b. Find the value of each model at $x = 15$.

7.

x	y
1	50
2	42
3	37
4	33
5	31
6	28
7	27

ANSWER:

a. $y = 50.11 - 12.06 \ln x$
b. 17.46

8.

x	y
2	8.6
4	7.2
6	6.4
8	5.8
10	5.4
12	5.0
14	4.7

ANSWER:

a. $y = 9.98 - 2 \ln x$
b. 4.56

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9.

x	y
1	40
2	49.9
3	55.8
4	59.9
5	63.2
6	65.8
7	68.1

ANSWER:

a. $y = 39.95 + 14.44 \ln x$
b. 79.04

10. **CHEMISTRY** A lab received a sample of an isotope of cobalt in 1999. The amount of cobalt in grams remaining per year is shown in the table below.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Cobalt (g)	877	769	674	591	518	454	398	349

a. Make a scatter plot of the data.
b. Find a logarithmic function to model the data. Let $x = 1$ represent 2000.
c. Predict the amount of cobalt remaining in 2020.

ANSWER:

a. $y = 922.18 - 259.08 \ln x$
b. 133 g

For Exercises 11-13, complete each step.
a. Find a logistic function to model the data.
b. Find the value of each model at $x = 25$.

11.

x	y
0	50
2	67
4	80
6	89
8	94
10	97
12	98
14	99

ANSWER:

a. $y = \dfrac{99.65}{1 + 0.99e^{-0.35x}}$
b. 99.63

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12.

x	y
1	3
2	5
3	7
4	8
5	13
6	16
7	19
8	20

ANSWER:

a. $y = \dfrac{24.63}{1 + 12.74e^{-0.52x}}$

b. 24.63

13.

x	y
3	21
6	25
9	28
12	31
15	33
18	34
21	35
24	35

ANSWER:

a. $y = \dfrac{36.16}{1 + 1.19e^{-0.16x}}$

b. 35.43

14. **CHEMISTRY** A chemistry is performing a titration in lab. To perform the titration, she uses a burette to add a basic solution of NaOH to a neutral solution. The table shows the pH of the solution as the NaOH is added.

NaOH (mL)	0	1	2	3	5	7.5	10
pH	10	10.4	10.6	11.0	11.3	11.5	11.5

a. Find a logistic function to model the data.

b. Use the model to predict the pH of the solution after 12 milliliters of NaOH have been added.

ANSWER:

a. $y = \dfrac{11.62}{1 + 0.17e^{-0.33x}}$

b. 11.36