

Name: _____ Key

5.2 EXTRA

$$1) \tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$$

EASY

$$\begin{aligned} &= \tan^2 x (\sec^2 x - 1) \\ &= \tan^2 x (\tan^2 x) \\ \tan^4 x &\stackrel{\checkmark}{=} \tan^4 x \end{aligned}$$

Factor!

MEDIUM

$$2) (\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$$

$$(\sec^2 x) (-\sin^2 x)$$

$$\frac{1}{\cos^2 x} \cdot \frac{(-\sin^2 x)}{1} =$$

$$\frac{-\sin^2 x}{\cos^2 x}$$

$$-\tan^2 x \stackrel{\checkmark}{=} -\tan^2 x$$

$$\begin{aligned} \cos^2 x + \sin^2 x &= 1 \\ \cos^2 x + \sin^2 x - 1 &= 0 \\ \cos^2 x - 1 &\stackrel{\cancel{\text{cancel}}}{=} \sin^2 x \end{aligned}$$

* I also saw ... since $1 + \tan^2 x = \sec^2 x$

$$\begin{aligned} &= \tan^2 (1 + \tan^2 x) - \tan^2 x \\ &= \tan^2 x + \tan^4 x - \tan^2 x \\ \tan^4 x &\stackrel{\checkmark}{=} \tan^4 x \end{aligned}$$

Name: _____

5.2 EXTRA

$$1) \tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$$

sines + cosines approach

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^4 x} - \frac{\sin^2 x}{\cos^4 x} (\cos^2 x)$$

$$= \frac{\sin^2 x - \sin^2 x \cdot \cos^2 x}{\cos^4 x}$$

$$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^4 x}$$

$$= \frac{\sin^2 x (\sin^2 x)}{\cos^4 x}$$

$$= \frac{\sin^4 x}{\cos^4 x} \quad \tan^4 x \stackrel{\checkmark}{=} \tan^4 x$$

$$2) (\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$$

I also saw ...

$$(\sec^2 x) (\cos^2 x - 1) =$$

$$\sec^2 x \cdot \cos^2 x - \sec^2 x =$$

$$1 - \underline{\sec^2 x} =$$

$$-\tan^2 x \stackrel{\checkmark}{=} -\tan^2 x$$

$$\begin{aligned} 1 + \tan^2 x &= \sec^2 x \\ -\sec^2 x &= -\sec^2 x \\ 1 + \tan^2 x - \sec^2 x &= 0 \\ \tan^2 x &\stackrel{\cancel{\text{cancel}}}{=} -\tan^2 x \\ 1 - \sec^2 x &= \tan^2 x \end{aligned}$$

Working the right side...

3) $\frac{1}{\csc x \cot x} = \sec x - \cos x$

$$= \frac{1}{\csc x} - \frac{\cos x}{1} \quad (\cancel{\csc x})$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \frac{\sin x}{\csc x} \cdot \frac{\sin x}{1}$$

$$= \frac{\tan x \cdot \sin x}{1}$$

$$= \frac{1}{\cot x} \cdot \frac{1}{\csc x}$$

$$\frac{1}{\csc x \cdot \cot x} \stackrel{?}{=} \frac{1}{\csc x \cdot \cot x}$$

4) $\sec \theta = \frac{\sec \theta - 1}{1 - \cos \theta} \quad (1 + \cos \theta)$

$$\frac{(1) \cancel{\sec \theta + \cos \theta \sec \theta - 1 - \cos \theta}}{1 - \cos^2 \theta}$$

$$\frac{\sec \theta + 1 - 1 - \cos \theta}{1 - \cos^2 \theta}$$

$$\frac{\sec \theta - \cos \theta}{1 - \cos^2 \theta} \quad \text{common denominator for numerator!}$$

$$\frac{\frac{1}{\cos \theta} - \frac{\cos \theta}{1} \quad (\cancel{\cos \theta})}{1 - \cos^2 \theta}$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} \quad *$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} \cdot \frac{1}{1 - \cos^2 \theta}$$

$$\frac{1}{3} = \frac{1}{3} \cdot \frac{1}{3}$$

$$\frac{1}{\cos \theta}$$

$$\sec \theta \stackrel{?}{=} \sec \theta$$

Remember...

Working the left

3) $\frac{1}{\csc x \cot x} = \sec x - \cos x$

$$\sin x \cdot \frac{\sin x}{\cos x} =$$

$$\frac{\sin^2 x}{\cos x} =$$

Identity
 $(1 - \cos^2 x) =$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} =$$

$$\sec x - \frac{\cos x \cdot \cos x}{\cos x} =$$

$$\sec x - \csc x \stackrel{?}{=} \sec x - \cos x$$

4) $\sec \theta = \frac{\sec \theta - 1}{1 - \cos \theta} \quad \text{I also saw...}$

$$= \frac{\sec \theta - 1}{(sec \theta) \left(1 - \frac{1}{\sec \theta} \right)}$$

$$= \frac{\sec \theta - 1}{\frac{\sec \theta - 1}{\sec \theta}}$$

$$= \frac{\sec \theta - 1}{1} \cdot \frac{\sec \theta}{\sec \theta - 1}$$

think...

$$\frac{4}{3}$$

$$4 \cdot \frac{4}{3}$$

$$\sec \theta \stackrel{?}{=} \sec \theta$$