

5.2 Verifying Trig Identities

$$1) \csc \theta \cot^2 \theta + \frac{1}{\sin \theta} = \csc^3 \theta$$

$$\frac{\cos^2 \theta}{\sin^3 \theta} + \frac{1}{\sin \theta (\sin^2 \theta)} =$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\sin^3 \theta} =$$

$$\frac{1}{\sin^3 \theta} =$$

$$(\csc^3 \theta) \checkmark = (\csc^3 \theta)$$

$$3) \frac{\cos x}{1 - \sin x} = \sec x + \tan x$$

$$\frac{\cos x + \cos x \sin x}{1 - \sin^2 x}$$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$\sec x + \tan x \checkmark$$

$$\frac{\cos x + \cos x \sin x}{\cos^2 x}$$

$$\frac{\cos x}{\cos^2 x} + \frac{\cos x \sin x}{\cos^2 x}$$

$$5) \frac{\cot^2 x}{\csc x} = \csc x - \sin x$$

$$\frac{\csc^2 x - 1}{\csc x} =$$

$$\csc x - \frac{1}{\csc x} =$$

$$(\csc x - \frac{1}{\csc x}) =$$

$$(\csc x - \sin x) \checkmark = \csc x - \sin x$$

$$2) \frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

$$\frac{\cancel{\sec^2 \theta}}{\sec^2 \theta} - \frac{1}{\sec^2 \theta} =$$

$$1 - \cos^2 \theta =$$

$$\sin^2 \theta \checkmark = \sin^2 \theta$$

$$4) \frac{\csc^2 x - \cot^2 x}{1 - \sin^2 x} = \sec^2 x$$

$$\cancel{\cos^2 x}$$

$$\frac{(1 + \cot^2 x) - \cot^2 x}{\cos^2 x}$$

$$\frac{1}{\csc^2 x}$$

$$\csc^2 x \checkmark = \csc^2 x$$

$$6) \sec^3 x = \sec x \tan^2 x + \sec x$$

$$= \sec (\tan^2 x + 1)$$

$$= \sec (\sec^2 x)$$

$$\sec^3 x \checkmark = \sec^3 x$$

$$7) \tan x \sin x = \sec x - \cos x$$

$$= \frac{1}{\cos x} - \frac{\cos x}{1} (\cos x)$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \frac{\sin x \sin x}{\cos x}$$

$$\tan x \cdot \sin x \stackrel{v}{=} \tan x \cdot \sin x$$

$$9) \frac{1 - \cos^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} =$$

$$\tan^2 \theta \stackrel{v}{=} \tan^2 \theta$$

$$11) \sec x \csc x (\tan x + \cot x) = \sec^2 x + \csc^2 x$$

$$\sec x (\csc x \tan x + \sec x \cot x) =$$

$$\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right) + \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) =$$

$$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} =$$

$$\sec^2 x + \csc^2 x \stackrel{v}{=} \sec^2 x + \csc^2 x$$

$$8) \tan x = \sin x \sec x$$

$$= \sin x \left(\frac{1}{\cos x}\right)$$

$$= \frac{\sin x}{\cos x}$$

$$\tan x \stackrel{v}{=} \tan x$$

$$10) \frac{(\sin x + \cos x)^2}{\sin x \cos x} = 2 + \sec x \csc x$$

$$(\sin x + \cos x)$$

$$10) \frac{(\sin x + \cos x)^2}{\sin x \cos x} = 2 + \sec x \csc x$$

$$\frac{(\sin^2 x) + 2 \sin x \cos x + \cos^2 x}{\sin x \cos x} =$$

$$\frac{2 \sin x \cos x + 1}{\sin x \cos x} =$$

$$\frac{2 \sin x \cos x}{\sin x \cos x} + \frac{1}{\sin x \cos x}$$

$$2 + \sec x \csc x \stackrel{v}{=} 2 + \sec x \csc x$$

$$12) \frac{(\cos x)}{(\cos x)} \frac{\cos x}{1 - \sin x} - \frac{1 + \sin x}{\cos x} \frac{(1 - \sin x)}{(1 - \sin x)} = 0$$

$$\frac{\cos^2 x - (1 - \sin^2 x)}{\cos x (1 - \sin x)} =$$

$$\frac{1}{\cos^2 x + \sin^2 x - 1} =$$

$$\frac{1}{\cos x (1 - \sin x)} =$$

$$0 \stackrel{v}{=} 0$$