Pre – Calculus
Section 5.3 Practice

Name key

If the problem says "F.A.S." please find all solutions on the interval $[0,2\pi)$. If it says "STE" please solve the equation.

1. $4\cos^2 x = 5 - 4\sin x$ [FAS]

$4(1-\sin^2 x) = 5 - 4\sin x$

$4\sin^2 x - 4\sin x + 1$

$(\sin x - \frac{2}{4})(\sin x - \frac{2}{4})$

$\sin x = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

2. $2\cos x + \sqrt{3} = 0$ [FAS]

$\cos x = -\frac{\sqrt{3}}{2}$

$\cos^{-1}(-\frac{\sqrt{3}}{2}) = x$

$\frac{5\pi}{6}, \frac{7\pi}{6}$

3. $\sec^2 x - 2 = 0$ [STE]

$+2 \quad +2$

$\sqrt{\sec^2} = \sqrt{2}$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi, \frac{7\pi}{4} + 2n\pi$

4. $4\cos x = 1 + 2\cos x$ [FAS]

$-2\cos x \quad -2\cos x$

$2\cos x = 1$

$\cos^{-1}(\frac{1}{2}) = x$

$\frac{\pi}{3}, \frac{5\pi}{3}$

5. $\csc^2 x - 2 = 0$ [FAS]

$+2 \quad +2$

$\csc x = \pm\sqrt{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

6. $\sqrt{3}\tan x - 1 = 0$ [FAS]

$+1 \quad +1$

$\tan x = \frac{1}{\sqrt{3}}$

$\tan^{-1}(\frac{\sqrt{3}}{3}) = x$

7. $2\sin^2 x + \sin x - 1 = 0$ [FAS]

$\sin^2 x + \sin x - 2 = 0$

$(\sin x + \frac{2}{2})(\sin x - \frac{1}{2}) = 0$

$(\sin x + 1)(\sin x - \frac{1}{2}) = 0$

$\sin^{-1}(-1) = x \quad \sin^{-1}(\frac{1}{2}) = x$

$\frac{3\pi}{2} \quad \frac{\pi}{6}, \frac{7\pi}{6}$

8. $\sin^2 x - 2\cos x - 2 = 0$ [STE]

$(1-\cos^2 x) - 2\cos x - 2 = 0$

$\cos^2 x + 2\cos x + 1 = 0$

$(\cos x + 1)(\cos x + 1) = 0$

$\cos x = -1$

π

9. $4\cos^2 x - 1 = 0$ [STE]

$+\frac{1}{4} \quad +\frac{1}{4}$

$\sqrt{\cos^2} = \sqrt{\frac{1}{4}}$

$\cos x = \pm\frac{1}{2}$

$\frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

10. $3\sin\theta + 5 = -2\sin\theta$ [FAS]

$+2\sin\theta^{-5} \quad +2\sin\theta^{-5}$

$5\sin\theta = -5$

$\sin^{-1}(-1) = \theta$

$\frac{3\pi}{2}$

11. $\sec^2 x - \sec x = 2$ [FAS]

$=0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2 \quad \sec x = -1$

$\cos x = \frac{1}{2} \quad \cos x = -1$

$\frac{\pi}{3}, \frac{5\pi}{3}, \pi$

12. $3\sqrt{3}\tan x = 3$ [STE]

$\frac{3\sqrt{3}}{3\sqrt{3}} \quad \frac{3}{3\sqrt{3}}$

$\tan x = \frac{\sqrt{3}}{3}$

$\frac{\pi}{6} + 2n\pi$,

$\frac{7\pi}{6} + 2n\pi$

13. $\sec x \csc x = 2\csc x$ [STE]

$-\sec x \csc x \quad -\sec x \csc x$

$0 = \csc x(2 - \sec x)$

$\csc x = \frac{0}{1} \quad \sec x = 2$

$\sin x = \text{undef.} \quad \cos x = \frac{1}{2}$

~~NO solution~~

$\frac{\pi}{3} + 2n\pi$,

$\frac{5\pi}{3} + 2n\pi$

14. $3\tan^3 x = \tan x$ [FAS]

$=0$

$\tan(3\tan^2 - 1) = 0$

$\tan x = 0 \quad \tan^2 = \frac{1}{3}$

$\tan = \pm\sqrt{\frac{1}{3}}$

$\tan = \pm\frac{\sqrt{3}}{3}$

$\pi, 0$

$\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

15. $2\cos^2 x + \sin x - 1 = 0$ [FAS]

$2(1 - \sin^2 x) + \sin x - 1 = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(\sin x - 2)(\sin x + 1) = 0$

$\frac{2}{2} \quad \frac{1}{2}$

$\sin x = 1 \quad \sin x = -\frac{1}{2}$

$\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

16. $2\sin^2 x = \sin x + 3$ [FAS]

$\sin^2 x - \sin x - 6 = 0$

$(\sin x - 3)(\sin x + 2) = 0$

$\frac{3}{2} \quad \frac{2}{2}$

$\sin x = \frac{3}{2} \quad \sin x = -1$

~~Undef.~~

$\frac{3\pi}{2}$

17. $\cos x = -\frac{1}{2}$ [STE]

$\frac{2\pi}{3} + 2n\pi, \quad \frac{4\pi}{3} + 2n\pi$