

5.3 practice (by skill)

Part 1: [Isolate the trigonometric expression & root]

1) Solve the equation.

$$2 + 3 \cos x - 5 = 0$$

$$-2 \quad +5 \quad +5$$

$$3 \cos x = 3$$

$$\cos x = 1$$

or 2π

$0 + 2n\pi$

2) Solve the equation.

$$0 = \pi \sec x + \pi$$

$$-\pi = \pi \sec x$$

$$\frac{-\pi}{\pi} = \frac{\pi \sec x}{\pi}$$

$$-1 = \sec x$$

$$-1 = \cos x$$

$\pi + 2n\pi$

3) Solve the equation.

$$5 \tan^3 x - 5 = 0$$

$$+5 \quad +5$$

$$5 \tan^3 x = 5$$

$$\sqrt[3]{5 \tan^3 x} = \sqrt[3]{5}$$

$$\tan x = 1$$

$\frac{\pi}{4} + 2n\pi$
 $\frac{5\pi}{4} + 2n\pi$

4) Find all solutions on the interval $[0, 2\pi)$

$$0 = 2 \sin x - 1$$

$$+1 \quad -1$$

$$\frac{1}{2} = \sin x$$

$\frac{\pi}{6}, \frac{5\pi}{6}$

5) Find all solutions on the interval $[0, 2\pi)$

$$3 \tan x - \sqrt{3} = 0$$

$$+\sqrt{3} \quad +\sqrt{3}$$

$$3 \tan x = \sqrt{3}$$

$$\tan x = \frac{\sqrt{3}}{3}$$

$\frac{\pi}{6}, \frac{7\pi}{6}$

6) Find all solutions on the interval $[0, 2\pi)$

$$3 \cos^2 x = 3$$

$$\sqrt{3 \cos^2 x} = \sqrt{3}$$

$$\cos x = \pm 1$$

$0, \pi$

7) Solve the equation.

$$-4 = -6 - \sqrt{3} \sec x$$

$$+6 \quad +6$$

$$2 = -\sqrt{3} \sec x$$

$$\frac{2}{-\sqrt{3}} = \frac{-\sqrt{3} \sec x}{-\sqrt{3}}$$

$$-\frac{\sqrt{3}}{2} = \cos x$$

$\frac{5\pi}{6} + 2n\pi$
 $\frac{7\pi}{6} + 2n\pi$

8) Find all solutions on the interval $[0, 2\pi)$

$$9 \tan^2 x - 3 = 0$$

$$+3 \quad +3$$

$$9 \tan^2 x = 3$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

9) Solve the equation.

$$3 \cos x = \cos x - 1$$

$$- \cos x \quad - \cos x$$

$$2 \cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$\frac{2\pi}{3} + 2n\pi$ and $\frac{4\pi}{3} + 2n\pi$

Part 2: [Factoring]

10) Find all solutions on the interval $[0, 2\pi)$

$$2 \cos^4 x - \cos^2 x = 0$$

$$\cos^2 x (2 \cos^2 x - 1) = 0$$

$$\cos^2 x = 0$$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$$2 \cos^2 x - 1 = 0$$

$$2 \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

11) Find all solutions on the interval $[0, 2\pi)$

$$3 \cot^4 x - 24 = \cot^2 x$$

$$3 \cot^4 x - \cot^2 x - 24 = 0$$

$$\cot^2 x (3 \cot^2 x - 1) - 24 = 0$$

$$(3 \cot^2 x - 1)(\cot^2 x + 8) = 0$$

$$3 \cot^2 x - 1 = 0$$

$$\cot^2 x = \frac{1}{3}$$

$$\cot x = \pm \frac{1}{\sqrt{3}}$$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

12) Find all solutions on the interval $[0, 2\pi)$

$$2 \sin^3 x = \sin x$$

$$2 \sin^3 x - \sin x = 0$$

$$\sin x (2 \sin^2 x - 1) = 0$$

$$\sin x = 0$$

$0, \pi$

$$2 \sin^2 x - 1 = 0$$

$$2 \sin^2 x = 1$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

13) Find all solutions on the interval $[0, 2\pi)$

$$\sqrt{3} \cos x \tan x - \cos x = 0$$

$$\cos x (\sqrt{3} \tan x - 1) = 0$$

$$\cos x = 0$$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{\pi}{6}, \frac{7\pi}{6}$$

14) Solve the equation.

$$-\tan^2 x = -2 \tan^4 x + 15$$

$$2 \tan^4 x - \tan^2 x - 15 = 0$$

$$\left(\tan^2 x - \frac{5}{2}\right) \left(\tan^2 x + 3\right) = 0$$

$$\tan^2 x = \frac{5}{2}$$

$$\tan^2 x = -3$$

$$\tan x = \pm \sqrt{\frac{5}{2}}$$

$$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

15) Find all solutions on the interval $[0, 2\pi)$

$$\sec x \csc^2 x = 2 \sec x$$

$$\sec x (\csc^2 x - 2) = 0$$

$$\sec x = 0$$

$$\csc x = \pm \sqrt{2}$$

no where!

$$\sqrt{\csc^2 x - 2}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} \cdot \sqrt{2}$$

$$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Part 3: [Rewrite to use a single trig function; Pyth Ident]

16) Find all solutions on the interval $[0, 2\pi)$

$$\cos^2 x + \sin^2 x = 1$$

$$-2 \sin^2 x = -\cos x - 1$$

$$-2(1 - \cos^2 x) + \cos x + 1 = 0$$

$$-2 \cos^2 x + \cos x - 1 = 0$$

OR

$$2 \cos^2 x - \cos x + 1 = 0$$

$$\cos^2 x - \cos x + 2 = 0$$

$$(\cos x - 2) (\cos x + 1) = 0$$

$$\cos x = 1$$

$$\cos x = -\frac{1}{2}$$

$$0$$

$$\frac{2\pi}{3}, \frac{4\pi}{3}$$

17) Find all solutions on the interval $[0, 2\pi)$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\sec^2 \theta + \tan \theta = 1$$

$$1 + \tan^2 \theta + \tan \theta = 1$$

$$\tan \theta (\tan \theta + 1) = 0$$

$$\tan \theta = 0$$

$$0, \pi$$

$$\tan \theta = -1$$

$$\frac{3\pi}{4}, \frac{7\pi}{4}$$

18) Find all solutions on the interval $[0, 2\pi)$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x + 5 \sin x - 7 = 0$$

$$(1 - \sin^2 x) + 5 \sin x - 7 = 0$$

$$\sin^2 x - 5 \sin x + 6 = 0$$

$$(\sin x - 3) (\sin x - 2) = 0$$

$$\sin x = 3$$

$$\sin x = 2$$

no solution!