Pre-Calculus Name: _____

Algebra 2 Review: Completing the Square

General Steps for Completing the Square:

**GOAL: Create a perfect square trinomial (ex: $x^2 + 6x + 9$) that will factor into $(x + 3)^2$

Example 1: Write each equation in vertex form: $(x - h)^2 = a(y - k)$ [or] $(y - k)^2 = a(x - h)$ by completing the square.

a) $y = x^2 + 6x + 8$

$y = (x^2+6x+\underline{9}) + 8 + \underline{-9}$

$y = (x+3)^2 - 1$

OR

$y - 8 + \underline{9} = (x^2+6x+\underline{9})$

$y + 1 = (x+3)^2$

b) $y = 2x^2 + 6x + 6$

$y - 6 + \underline{4.50} = 2(x^2+3x+\underline{2.25})$

$\frac{(y - 1.50)}{2} = \frac{2(x+1.5)^2}{2}$

$\frac{1}{2}(y-1.50) = (x+1.5)^2$

c) $y = 2x^2 - 10x + 5$

$y - 5 + \underline{12.50} = 2(x^2-5x+\underline{6.25})$

$\frac{(y + 6.50)}{2} = \frac{2(x-2.5)^2}{2}$

$\frac{1}{2}(y+6.50) = (x-2.5)^2$

d) $y = -\frac{1}{4}x^2 + 3x + 6$

$y - 6 + \underline{-9} = -\frac{1}{4}(x^2-12x+\underline{36})$

$(-4)(y-15) = -\frac{1}{4}(x-6)^2(-4)$

$-4(y-15) = (x-6)^2$

e) $3y^2 + 6y + 15 = 12x$

$3(y^2+2y+\underline{1}) = 12x - 15 + \underline{3}$

$3(y+1)^2 = 12x - 12$

$\frac{3(y+1)^2}{3} = \frac{12(x-1)}{3}$

$(y+1)^2 = 4(x-1)$

Example 2: Complete the square for both x and y to write each equation in vertex form:

$$(x-h)^2+(y-k)^2=1 \text{ [or] } \frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

a) $4x^2+y^2-24x+4y+24=0$

$4x^2-24x+\_\_+y^2+4y+4=-24+\_+4$

$4(x^2-6x+9)+(y+2)^2=-20+36$

$\frac{4(x-3)^2}{16}+\frac{(y+2)^2}{16}=\frac{16}{16}$

$\frac{(x-3)^2}{4}+\frac{(y+2)^2}{16}=1$

b) $x^2+4y^2+4x-40y+103=0$

$x^2+4x+4+4(y^2-10y+25)=-103+4+100$

$(x+2)^2+4(y-5)^2=1$

c) $x^2+y^2-12x+10y+12=0$

$x^2-12x+36+y^2+10y+25=-12+36+25$

$(x-6)^2+(y+5)^2=49$

d) $25x^2-16y^2+100x+96y=144$

$25(x^2+4x+4)-16(y^2-6y+9)=144+100+-144$

$\frac{25(x+2)^2}{100}-\frac{16(y-3)^2}{100}=\frac{100}{100}$

$\frac{(x+2)^2}{4}-\frac{4(y-3)^2}{25}=1$

e) $2x^2-3y^2-12x=36$

$2(x^2-6x+9)-3y^2=36+18$

$\frac{2(x-3)^2}{54}-\frac{3y^2}{54}=\frac{54}{54}$

$\frac{2(x-3)^2}{54}-\frac{y^2}{18}=1$

Completing the Square Homework

Complete the square for the variable(s) in each equation.

1) $y = x^2 + 18x - 9$

$y = (x^2+18x+\underline{81}) - 9 + \underline{-81}$

$y = (x+9)^2 - 90$

OR

$y + 90 = (x+9)^2$

2) $y = 2x^2 - 12x + 17$

$y = 2(x^2-6x+\underline{9}) + 17 + \underline{-18}$

$y = 2(x-3)^2 - 1$

OR

$\frac{1}{2}(y+1) = (x-3)^2$

3) $y = -3x^2 + 24x$

$y = -3(x^2-8x+\underline{16}) + \underline{48}$

$y = -3(x-4)^2 + 48$

OR

$\frac{-1}{3}(y-48) = (x-4)^2$

4) $y^2 - 4y + 4x + 4 = 0$

$(y^2-4y+\underline{4}) = -4x - 4 + \underline{4}$

$(y-2)^2 = -4x$

5) $3x^2 + 3y^2 - 6x + 12y = 0$

$3(x^2-2x+\underline{1}) + 3(y^2+4y+\underline{4}) = 0 + \underline{3} + \underline{12}$

$\frac{3(x-1)^2}{15} + \frac{3(y+2)^2}{15} = \frac{15}{15}$

$\frac{(x-1)^2}{5} + \frac{(y+2)^2}{5} = 1$

6) $4x^2 - y^2 - 4y + 8x - 4 = 0$

$4(x^2+2x+\underline{1}) - (y^2+4y+\underline{4}) = 4 + \underline{4} + \underline{-4}$

$\frac{4(x+1)^2}{4} - \frac{(y+2)^2}{4} = \frac{4}{4}$

$(x+1)^2 - \frac{(y+2)^2}{4} = 1$

7) $x^2 + 4x + y^2 - 2y = 0$

$(x^2+4x+\underline{4}) + (y^2-2y+\underline{1}) = 0+\underline{4}+\underline{1}$

$(x+2)^2 + (y-1)^2 = 5$

8) $x^2 - 4x = y + 4$

$(x^2-4x+\underline{4}) = y+4+\underline{4}$

$(x-2)^2 = y+8$

OR

$y = (x-2)^2 - 8$

9) $x^2 + 3y^2 + 8x - 6y = 5$

$(x^2+8x+\underline{16}) + 3(y^2-2y+\underline{1}) = 5+\underline{16}+\underline{3}$

$\frac{(x+4)^2}{24} + \frac{3(y-1)^2}{24} = \frac{24}{24}$

$\frac{(x+4)^2}{24} + \frac{(y-1)^2}{8} = 1$

10) $2x^2 + 2y^2 + 8x + 7 = 0$

$2(x^2+4x+\underline{4}) + 2y^2 = -7+\underline{8}$

$\frac{2(x+2)^2}{2} + \frac{2y^2}{2} = \frac{1}{2}$

$(x+2)^2 + y^2 = \frac{1}{2}$