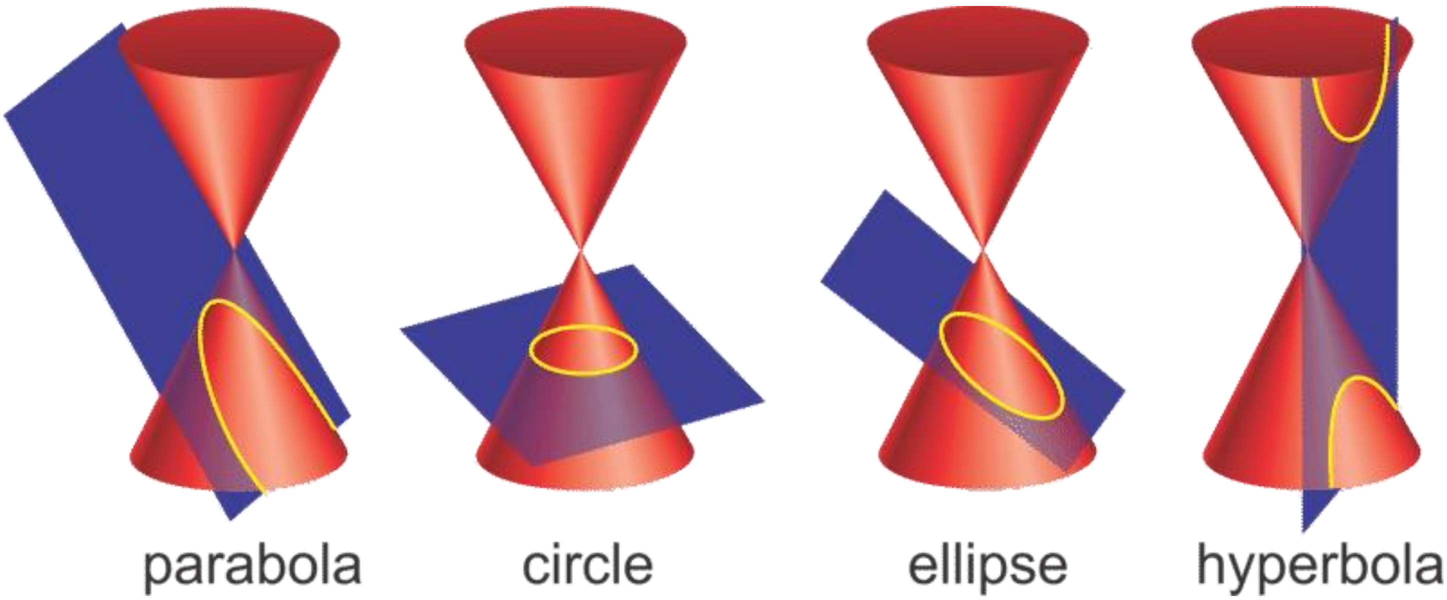
Name:______________________________

Pre-Calculus

Unit C: Conic Sections (Chapter 7)

Fall 2019


parabola
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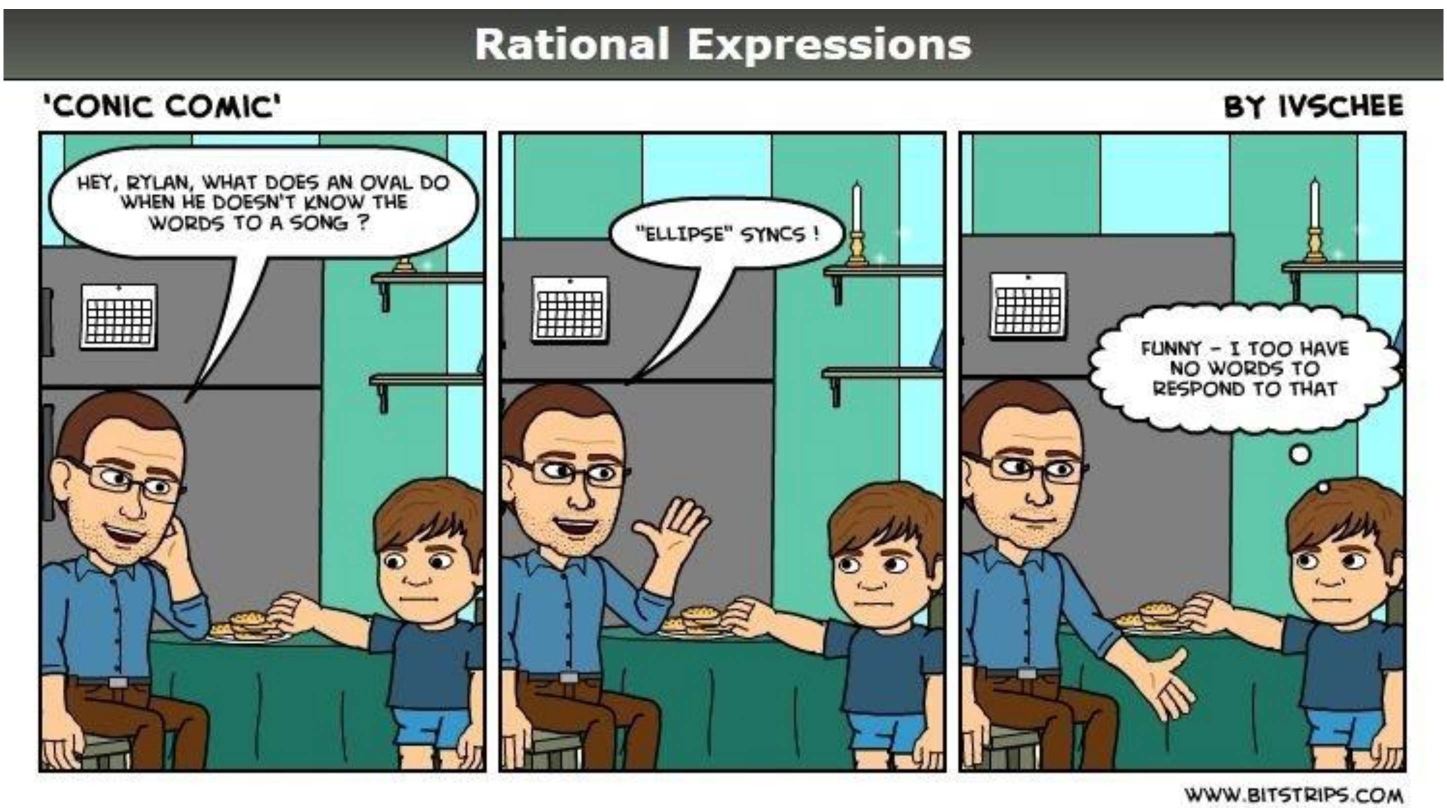
Rational Expressions
'CONIC COMIC'
BY IVSCHEE
HEY, RYLAN, WHAT DOES AN OVAL DO WHEN HE DOESN'T KNOW THE WORDS TO A SONG ?
"ELLIPSE" SYNCS !
FUNNY - I TOO HAVE NO WORDS TO RESPOND TO THAT
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Pre-Calculus Name: ________________________________

Algebra 2 Review: Completing the Square

General Steps for Completing the Square:

**GOAL: Create a perfect square trinomial (ex: $x^2 + 6x + 9$) that will factor into $(x + 3)^2$

Example 1: Write each equation in vertex form: $\boldsymbol{(x - h)^2 = a(y - k)}$ [or] $\boldsymbol{(y - k)^2 = a(x - h)}$ by completing the square.

a) $y = x^2 + 6x + 8$

b) $y = 2x^2 + 6x + 6$

c) $y = 2x^2 - 10x + 5$

d) $y = -\frac{1}{4}x^2 + 3x + 6$

e) $3y^2 + 6y + 15 = 12x$

Example 2: Complete the square for both x and y to write each equation in vertex form:

$$\boldsymbol{(x - h)^2 + (y - k)^2 = 1} \text{ [or] } \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

a) $4x^2 + y^2 - 24x + 4y + 24 = 0$

b) $x^2 + 4y^2 + 4x - 40y + 103 = 0$

c) $x^2 + y^2 - 12x + 10y + 12 = 0$

d) $25x^2 - 16y^2 + 100x + 96y = 144$

e) $2x^2 - 3y^2 - 12x = 36$

Complete the square for the variable(s) in each equation.

1) $y = x^2 + 18x - 9$

2) $y = 2x^2 - 12x + 17$

3) $y = -3x^2 + 24x$

4) $y^2 - 4y + 4x + 4 = 0$

5) $3x^2 + 3y^2 - 6x + 12y = 0$

6) $4x^2 - y^2 - 4y + 8x - 4 = 0$

7) $x^2 + 4x + y^2 - 2y = 0$

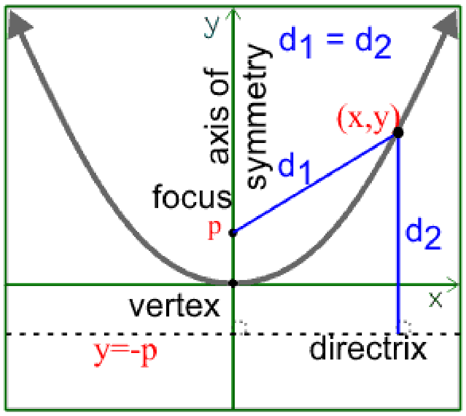
8) $x^2 - 4x = y + 4$

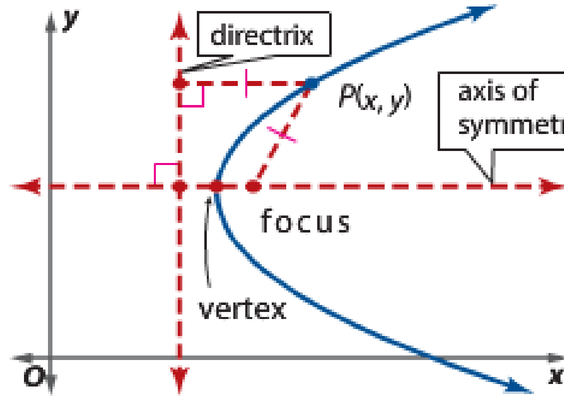
9) $x^2 + 3y^2 + 8x - 6y = 5$

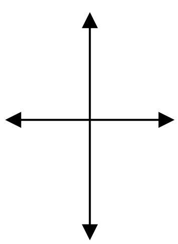
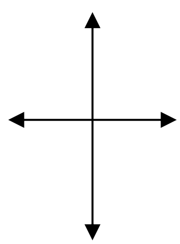
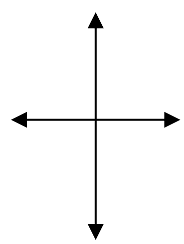
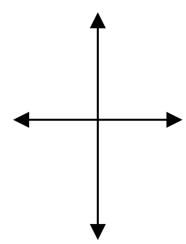
10) $2x^2 + 2y^2 + 8x + 7 = 0$

Pre-Calculus
Section 7.1 Notes: Parabolas

Objectives:
- Analyze and graph equations of parabolas
- Write equations of parabolas

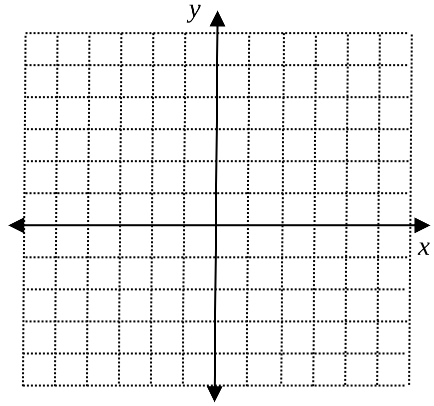




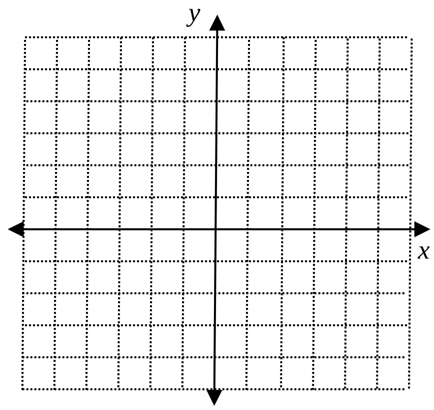
Conic Section		
Equation		
Graph	 	 
Axis of Symmetry		
Vertex		
Focus		
Directrix		

Example 1: identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

a) $(y - 3)^2 = -8(x + 1)$



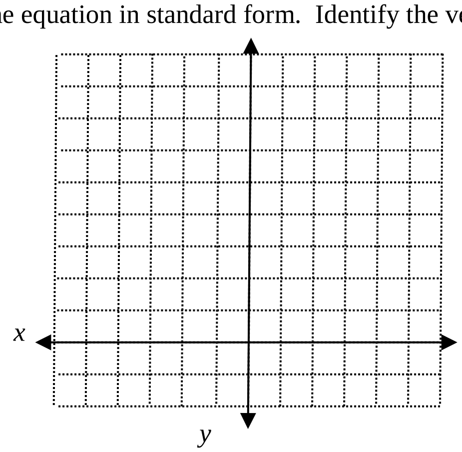
b) $(x + 1)^2 = -4(y - 2)$



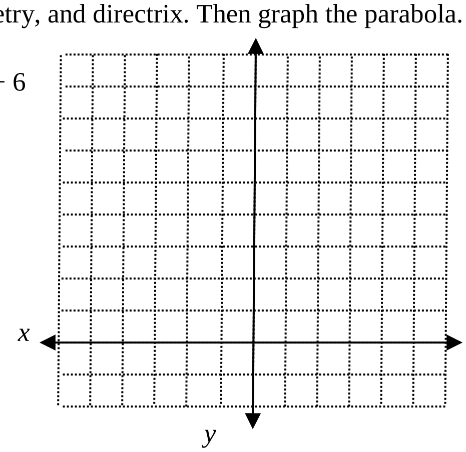
Example 2: The parabolic mirror for the California Institute of Technology's Hale telescope at Mount Palomar has a shape modeled by $y^2 = 2668x$, where x and y are measured in inches. What is the focal length of the mirror?

Example 3: Write the equation in standard form. Identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

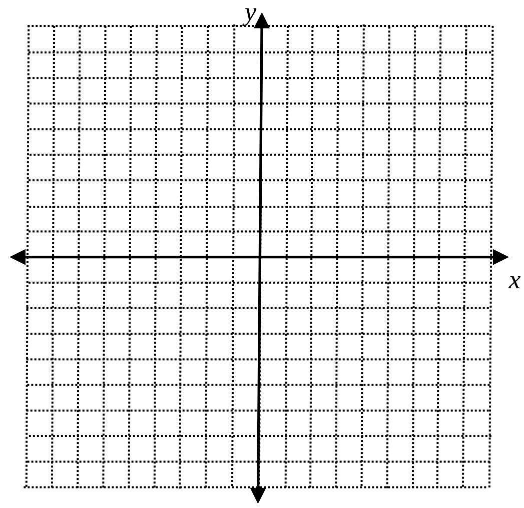
a) $x^2 - 8x - y = -18$



b) $y = -\frac{1}{2}x^2 + 3x + 6$

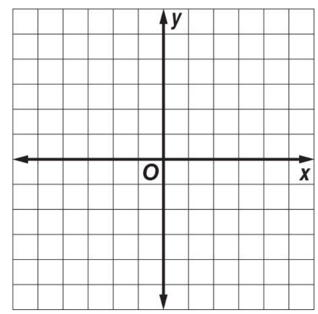


c) $y^2 + 16x = 55 - 6y$

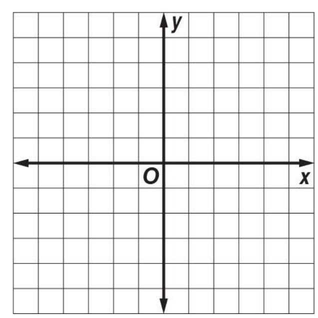


Example 4: Write an equation for and graph a parabola with the given characteristics.

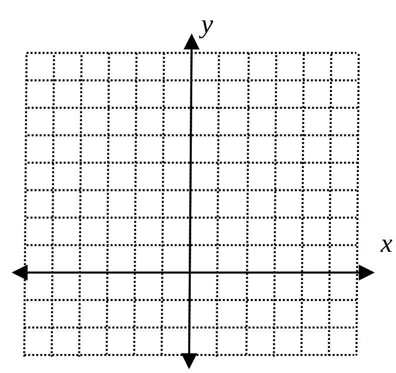
a) focus (2, 1) and vertex (−5, 1)



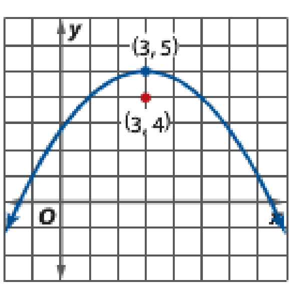
b) vertex (3, −2), directrix $y = -1$



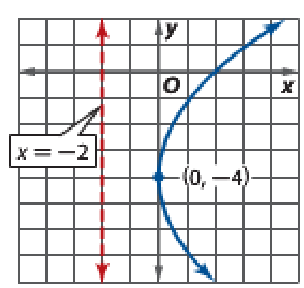
c) focus (−1, 7), opens up, contains (3, 7)



d) Write an equation for the graph:



e) Write an equation for the graph:



Pre-Calculus
Section 7.1: Parabolas Homework

For numbers 1 & 2, identify the vertex, focus, axis of symmetry, and directrix for each equation. Then graph the parabola.

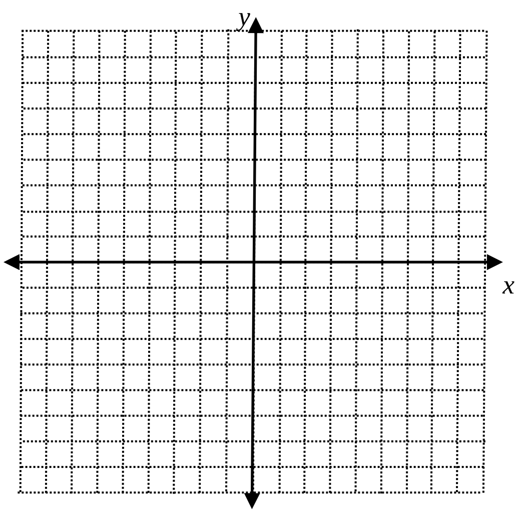
1. $(x - 1)^2 = 8(y - 2)$

Vertex: _______________

Focus: ________________

Axis of symmetry: ________________

Directrix: ____________________



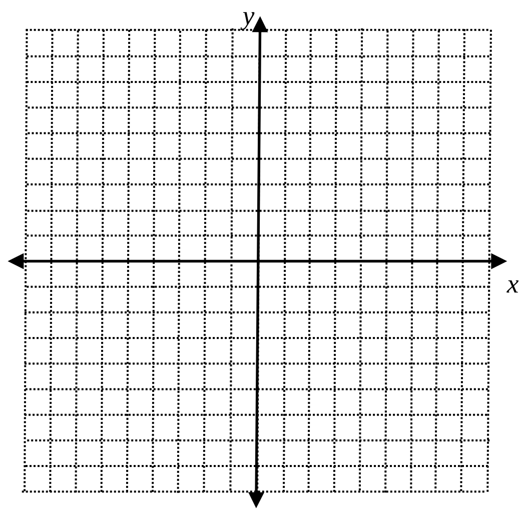
2. $y^2 + 6y + 9 = 12 - 12x$

Vertex: _______________

Focus: ________________

Axis of symmetry: ________________

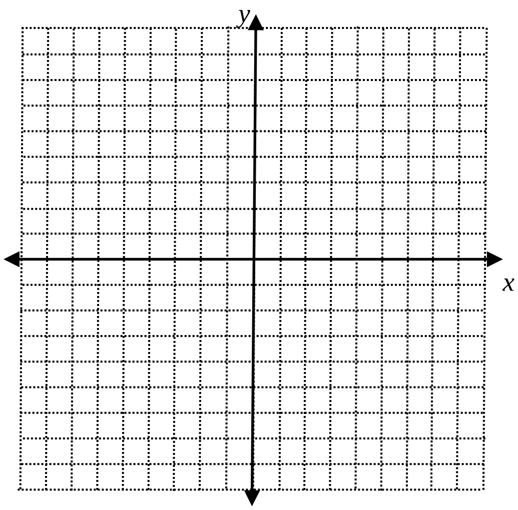
Directrix: ____________________



3. Write $x^2 + 8x = -4y - 8$ in standard form. Identify the vertex, focus, axis of symmetry, and directrix.

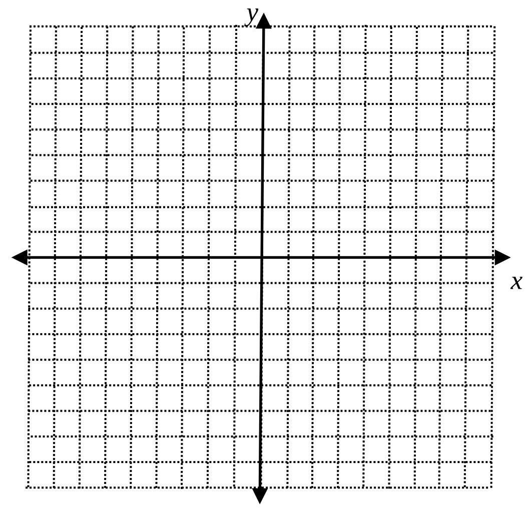
For numbers 4 & 5, write an equation for and graph a parabola with the given characteristics.

4. vertex (–2, 4); focus (–2, 3)



Equation: ________________________________

5. focus (2, 1); opens right; contains (8, –7)



Equation: ________________________________

6. Suppose the receiver in a parabolic dish antenna is 2 feet from the vertex and is located at the focus. Assume that the vertex is at the origin and that the dish is pointed upward. Find an equation that models a cross section of the dish.

7. The figure shows a parabolic reflecting mirror. A cross section of the mirror can be modeled by $x^2 = 16y$, where the values of x and y are measured in inches. Find the distance from the vertex to the focus of this mirror.

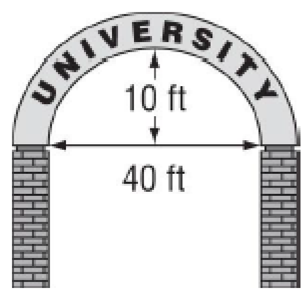
8. The cheerleaders at the high school basketball game launch T-shirts into the stands after a victory. The launching device propels the shirts into the air at an initial velocity of 32 feet per second. A shirt's distance y in feet above the ground after x seconds can be modeled by $y = -16x^2 + 32x + 5$.

a) Write the equation in standard form.

b) What is the maximum height that a T-shirt reaches?

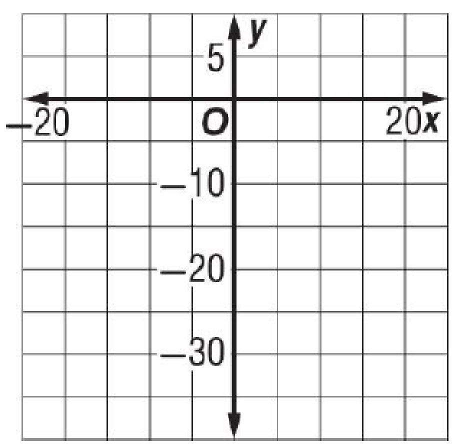
9. A flashlight contains a parabolic mirror with a bulb in the center as a light source and focus. If the width of the mirror is 4 inches at the top and the height to the focus is 0.5 inch, find an equation of the parabolic cross section.

10. The entrance to a college campus has a parabolic arch above two columns as shown in the figure.

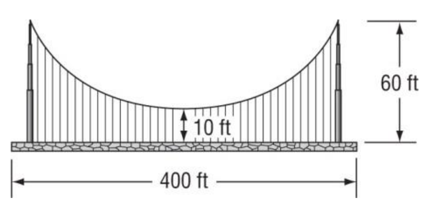


a) Write an equation that models the parabola.

b) Graph the equation.



11. The cable for a suspension bridge is in the shape of a parabola. The vertical supports are shown in the figure.



a) Write an equation for the parabolic cable.

b) Find the length of a supporting wire that is 100 feet from the center.

For numbers 12 – 15, determine the orientation of each parabola.

12. directrix $y = 4$, $p = -2$

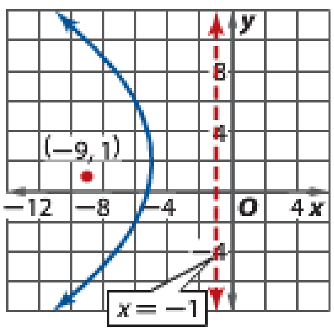
13. $y^2 = -8(x - 6)$

14. vertex (–5, 1) , focus (–5, 3)

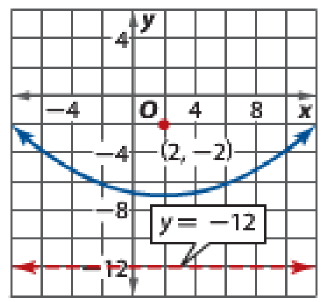
15. focus (7, 10), directrix $x = 1$

For numbers 16 – 17, write the equation for each parabola.

16.

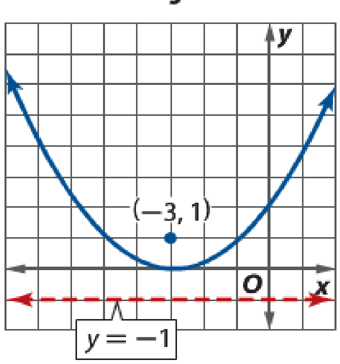


17.

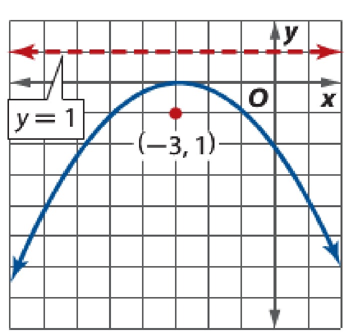


19. Abigail and Jaden are graphing $x^2 + 6x - 4y + 9 = 0$. Is either of them correct? Explain your reasoning.

Abigail

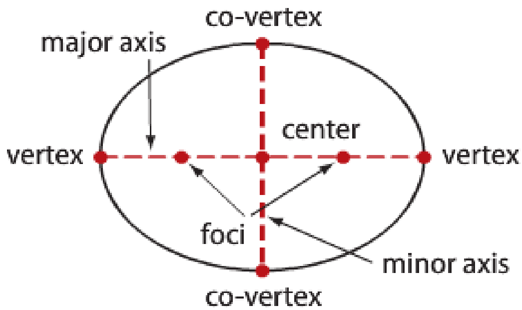


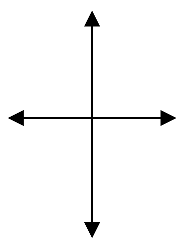
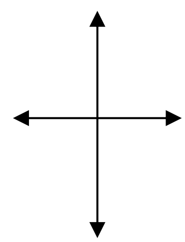
Jaden



Pre-Calculus
Section 7.2 Notes: Ellipses and Circles

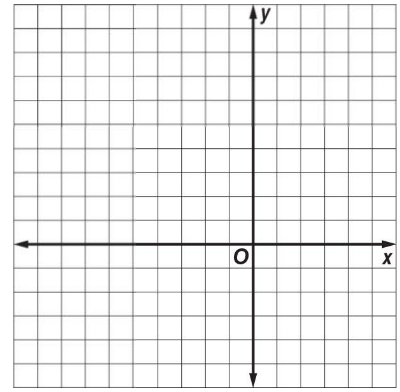
Objectives:
- Analyze the graphs of ellipses and circles
- Use equations to identify ellipses and circles



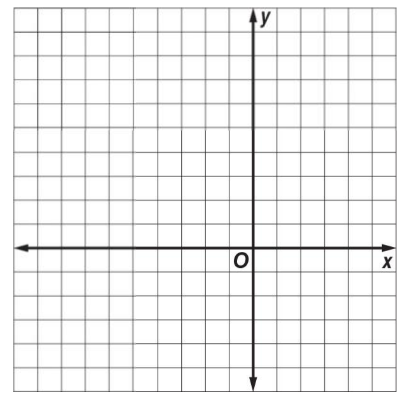
Conic Section		
Equation		
Graph		
Center		
Co-vertices		
Vertices		
Major Axis		
Minor Axis		
Foci		

Example 1: Graph the ellipse given by each equation.

a) $\frac{(x+2)^2}{9}+\frac{(y-1)^2}{4}=1$



b) $4x^2 + 24x + y^2 - 10y - 3 = 0$



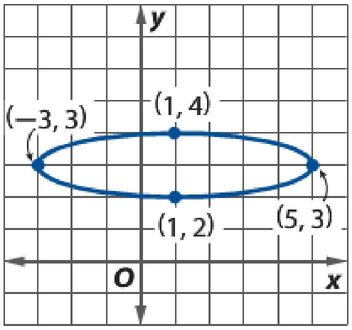
Example 2: Write an equation for an ellipse with the following characteristics:

a) a major axis from (5, –2) to (–1, –2) and a minor axis from (2, 0) to (2, –4).

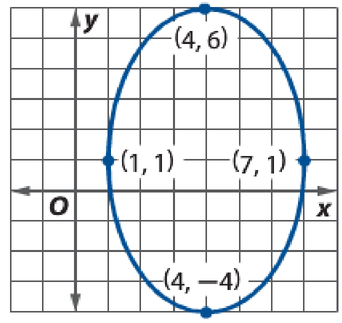
b) vertices at (3, –4) and (3, 6) and foci at (3, 4) and (3, –2).

c) co-vertices at (–8, 6) and (4, 6) and major axis of length 18.

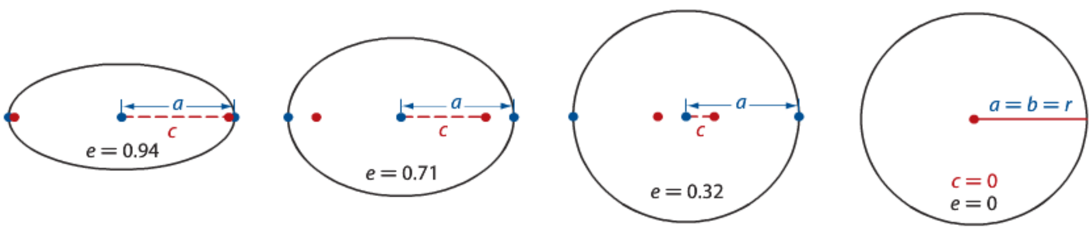
d) Write an equation for the ellipse shown.

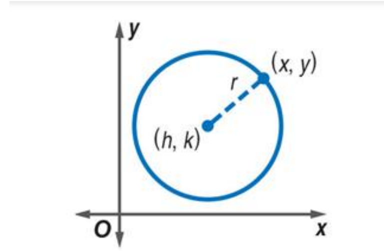


e) Write an equation for the ellipse shown.



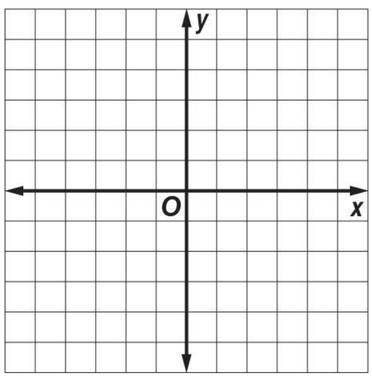
Eccentricity	



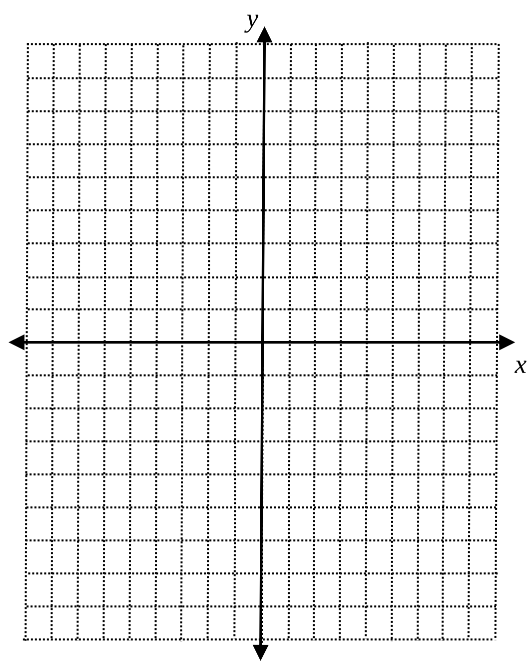
Conic Section	
Equation	
Radius	
Center	

Example 3: Graph the circle given by the equation.

a) $(x-4)^2+(y+1)^2=4$



b) $x^2-4x+y^2+6y=-9$



Pre-Calculus

Section 7.2: Ellipses and Circles Homework

For numbers 1 & 2, graph the ellipse given by each equation.

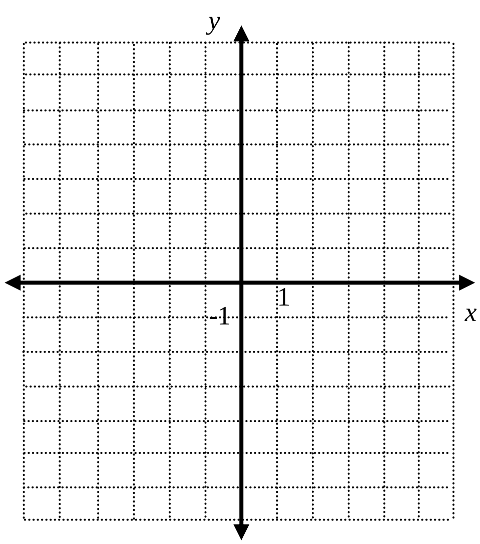
1. $4x^2 + 9y^2 - 8x - 36y + 4 = 0$

Equation: ____________________________

Foci: ________________________________

Vertices: _____________________________

Co-Vertices: __________________________



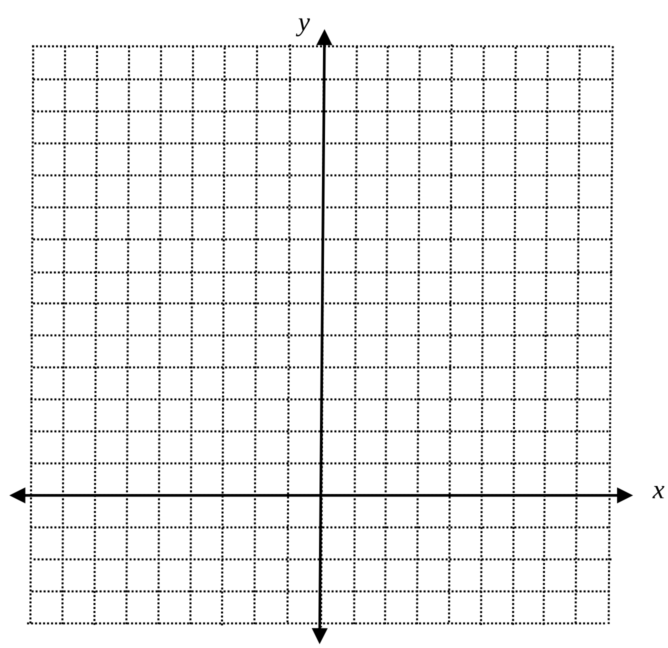
2. $25x^2 + 9y^2 - 50x - 90y + 25 = 0$

Equation: ____________________________

Foci: ________________________________

Vertices: _____________________________

Co-Vertices: __________________________



For numbers 3 & 4, write an equation for the ellipse with each set of characteristics.

3. vertices (–12, 6), (4, 6); foci (–10, 6), (2, 6)

4. foci $(-2, 1)$, $(-2, 7)$; length of major axis 10 units

For numbers 5 – 8, write each equation in standard form. Identify the related conic.

5. $y^2 - 4y = 4x + 16$

6. $4x^2 - 32x + 3y^2 - 18y = -55$

7. $x^2 + y^2 - 8x - 24y = 9$

8. $x^2 + y^2 + 20x - 10y + 4 = 0$

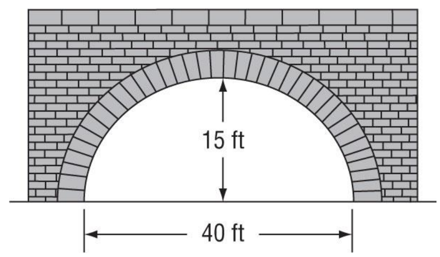
9. A semi-elliptical arch is used to design a headboard for a bed frame. The headboard will have a height of 2 feet at the center and a width of 5 feet at the base. Where should the craftsman place the foci in order to sketch the arch (In other words, find the location of the foci)?

10. A whispering gallery at a museum is in the shape of an ellipse. The room is 84 feet long and 46 feet wide.

a) Write an equation modeling the shape of the room. Assume that it is centered at the origin and that the major axis is horizontal.

b) Find the location of the foci.

11. The entrance to a tunnel is in the shape of half an ellipse as shown in the figure. (You may assume that the ellipse is centered at the origin)

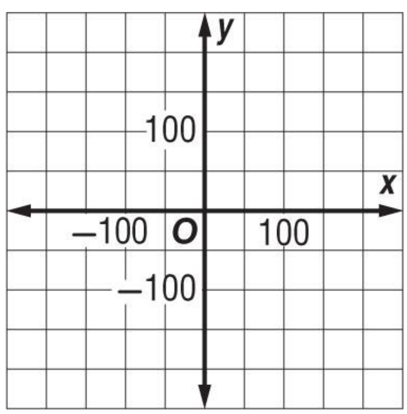


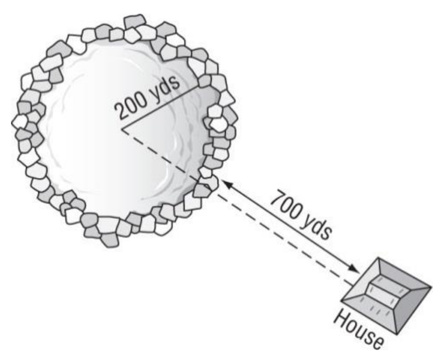
a) Write an equation that models the ellipse.

b) Find the height of the tunnel 10 feet from the center.

12. A circular retention pond is getting larger by overflowing and flooding the nearby land at a rate that increases the radius 100 yards per day, as shown below.

a) Graph the circle that represents the water, and find the distance from the center of the pond to the house.



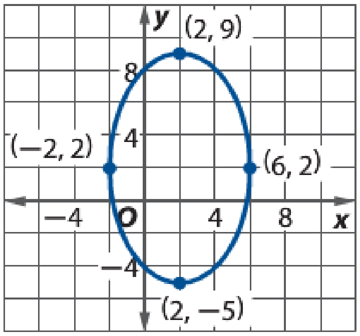


b) If the pond continues to overflow at the same rate, how many days will it take for the water to reach the house?

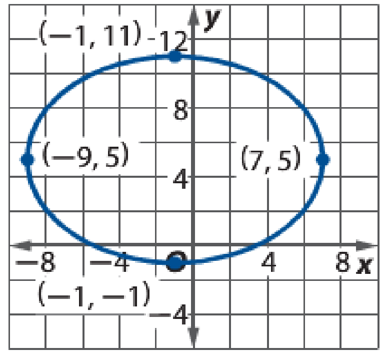
c) Write an equation for the circle of water at the current time and an equation for the circle when the water reaches the house.

For numbers 13 – 14, write an equation for each ellipse.

13.

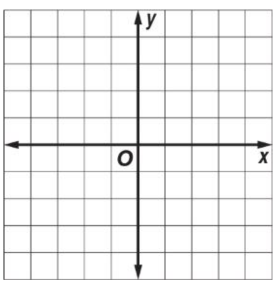


14.

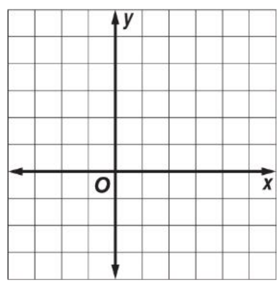


For numbers 15 – 18, graph the circle.

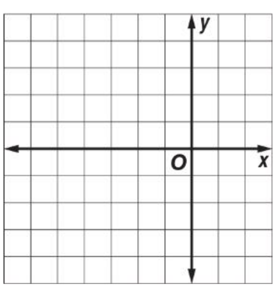
15. $x^2 + y^2 = 16$



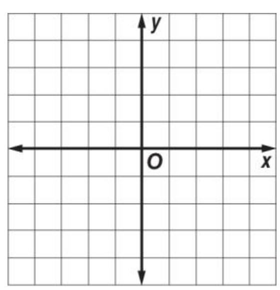
16. $(x - 2)^2 + (y - 1)^2 = 9$



17. $(x + 2)^2 + y^2 = 16$

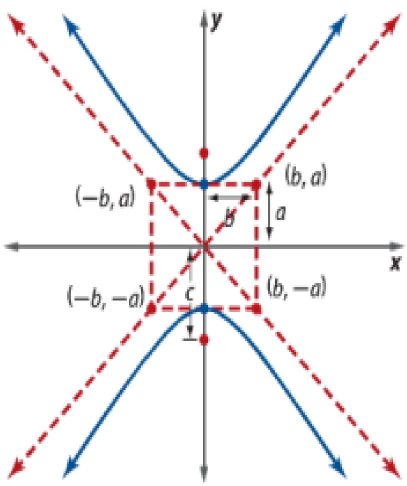


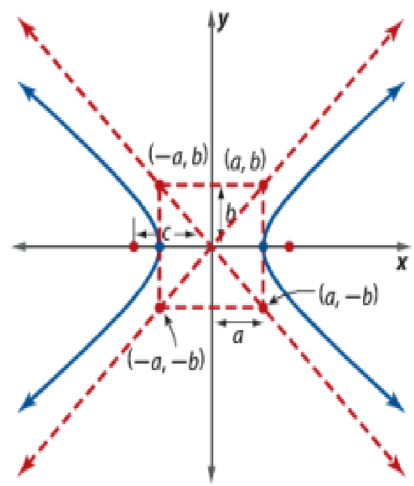
18. $x^2 + (y - 1)^2 = 9$



Pre-Calculus
Section 7.3 Notes: Hyperbolas

<u>Objectives:</u>
- Analyze and graph equations of hyperbolas
- Use equations to identify types of conic sections





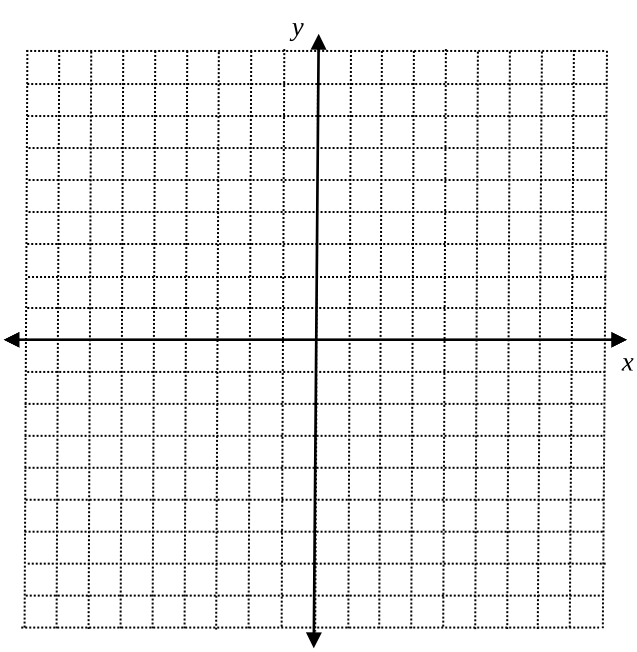
Conic Section		
Equation		
Asymptotes (Equation)		
Center		
Vertices		
Conjugate Axis		
Foci		

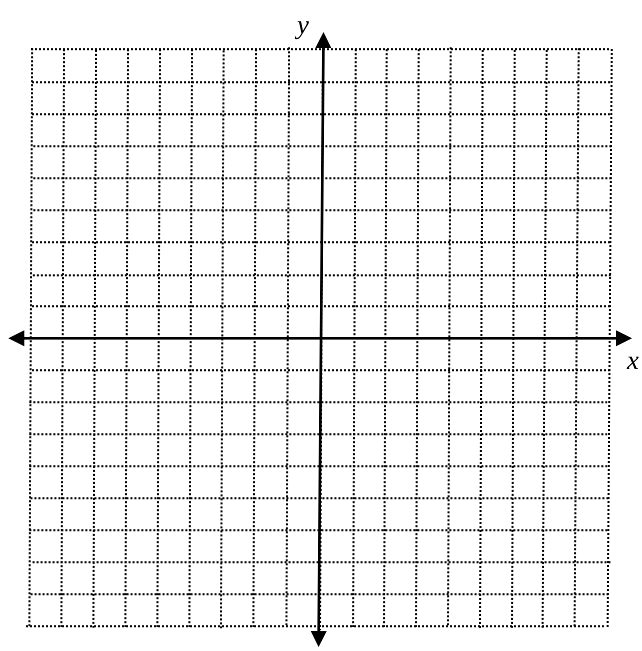
What to know	
Approach to Graphing Hyperbolas	

1: Graph the hyperbola given by the equations.

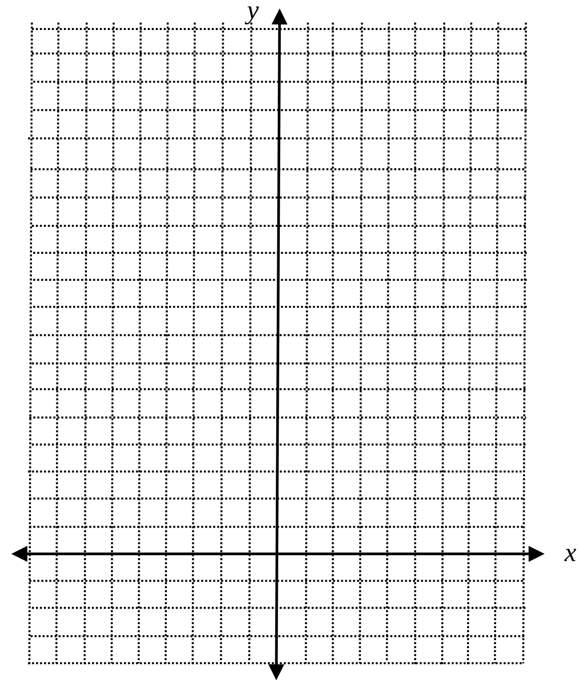
a) $\frac{x^2}{49} - \frac{y^2}{81} = 1$

b) $\frac{(y+4)^2}{4} - \frac{(x-2)^2}{9} = 1$.





Example 2: Graph the hyperbola given by $4x^2 - y^2 + 24x + 4y = 28$.



Example 3: Write an equation for the hyperbola with the given characteristics.

a) foci (1, –5) and (1, 1) and transverse axis length of 4 units.

b) vertices (–3, 10) and (–3, –2) and conjugate axis length of 6 units**.**

c) center (–7, 2); asymptotes $y = \pm\frac{7}{5}x + \frac{59}{5}$, transverse axis length of 10 units and opens horizontally

Determining the type of conic			
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$			
Circle	Ellipse	Parabola	Hyperbola

Example 5: Write each equation in standard form. Identify the related conic.

a) $9x^2 + 4y^2 + 8y - 32 = 0$

b) $x^2 + 4x - 4y + 16 = 0$

c) $x^2 + y^2 + 2x - 6y - 6 = 0$

Example 6: Use the discriminant to identify the conic section in the given equation.

a) $2x^2 + y^2 - 2x + 5xy + 12 = 0$

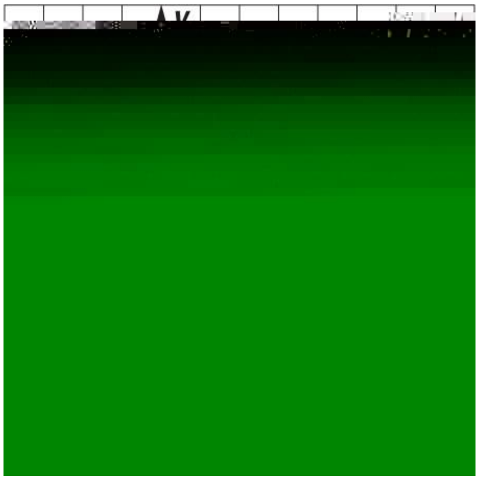
b) $4x^2 + 4y^2 - 4x + 8 = 0$

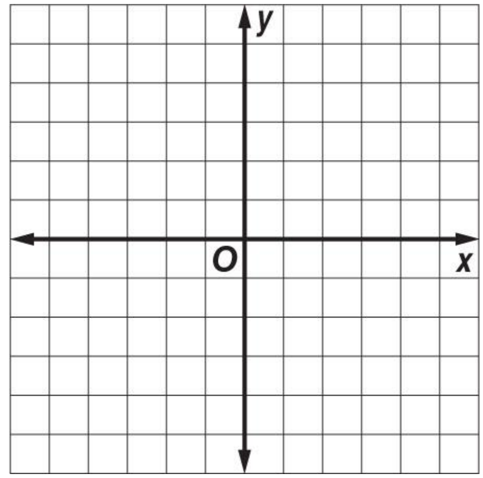
c) $2x^2 + 2y^2 - 6y + 4xy - 10 = 0$.

For numbers 1 & 2, graph the hyperbola given by each equation.

1. $x^2 - 4y^2 - 4x + 24y - 36 = 0$

2. $\frac{y^2}{16} - \frac{(x-1)^2}{4} = 1$





For numbers 3 – 8, write an equation for the hyperbola with the given characteristics.

3. vertices (–10, 6), (4, 6); foci (–12, 6), (6, 6)

4. foci (0, 6), (0, –4); length of transverse axis 8 units

5. foci (–1, 9), (–1, –7); conjugate axis length of 14 units

6. vertices (–1, 9), (–1, 3) asymptotes $y = \pm\frac{3}{7}x + \frac{45}{7}$

7. center (0, –5); asymptotes $y = \pm\frac{\sqrt{19}}{6}x - 5,$ conjugate axis length of 12 units

For numbers 8 – 11, use the discriminant to identify each conic section.

8. $5x^2 + xy + 2y^2 - 5x + 8y + 9 = 0$

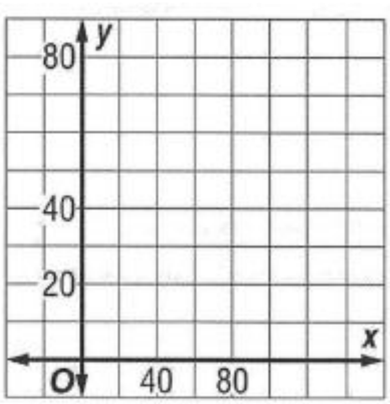
9. $16x^2 - 4y^2 - 8x - 8y + 1 = 0$

10. $4x^2 + 8xy + 4y^2 + x + 11y + 10 = 0$

11. $2x^2 + 4y^2 - 3x - 6y + 2 = 0$

12. **EARTHQUAKES** The epicenter of an earthquake lies on a branch of the hyperbola represented by $\frac{(x-50)^2}{1600} - \frac{(y-35)^2}{2500} = 1$, where the seismographs are located at the foci.

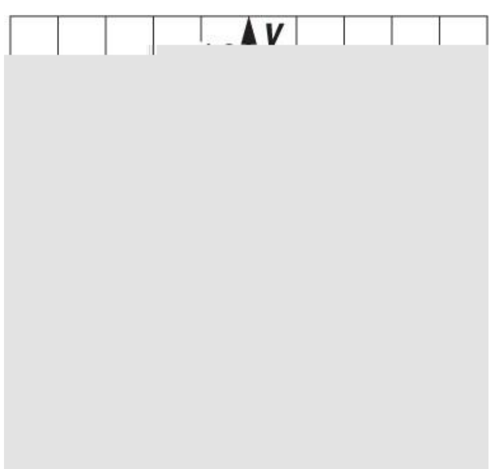
a) Graph the hyperbola.



b) Find the locations of the seismographs.

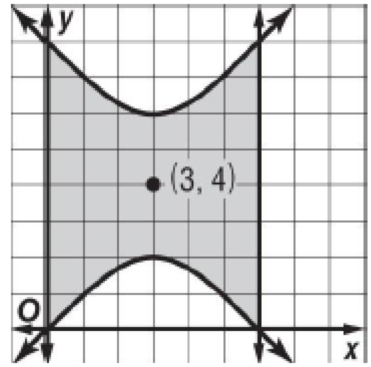
13. A lamp projects light onto a wall in the shape of a hyperbola. The edge of the light can be modeled by $\frac{y^2}{196} - \frac{x^2}{121} = 1$.

a) Graph the hyperbola.



b) Write the equations of the asymptotes.

14. A grassy play area is in the shape of a hyperbola, as shown.



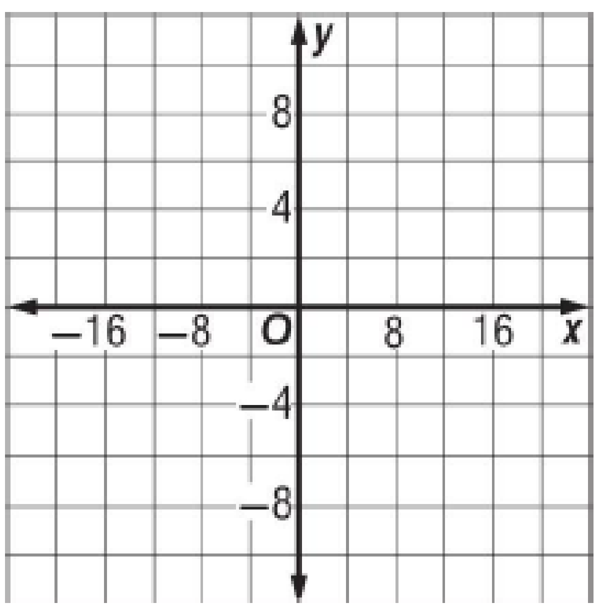
a) Write an equation that models the curved sides of the play area.

b) If each unit on the coordinate plane represents 3 feet, what is the narrowest vertical width of the play area?

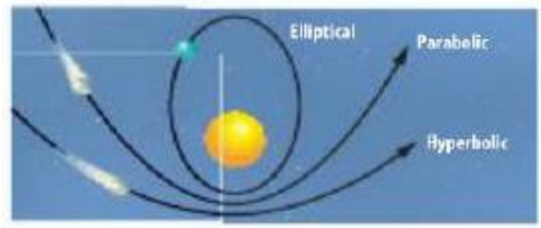
15. The path of the shadow cast by the tip of a sundial is usually a hyperbola.

a) Write two equations of the hyperbola in standard form if the center is at the origin, given that the path contains a transverse axis of 24 millimeters with one focus 14 millimeters from the center.

b) Graph one hyperbola.



16. While each of the planets in our solar system move around the Sun in elliptical orbits, comets may have elliptical, parabolic, or hyperbolic orbits where the center of the sun is a focus.



The paths of three comets are modeled below, where the values of x and y are measured in gigameters. Use the discriminant to identify each conic.

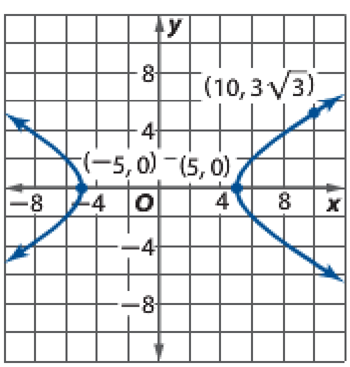
a) $3x^2 - 18x - 580850 = 4.84y^2 - 38.72y$

b) $-360x - 8y = -y^2 - 1096$

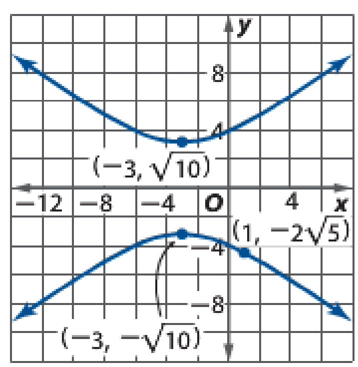
c) $-24.88y + x^2 = 6x - 3.11y^2 + 412341$

For numbers 17 & 18, write an equation for each hyperbola.

17.



18.



Equation Sheet: Conic Sections

Parabolas

	Horizontal Direction of Opening	Vertical Direction of Opening
Equation	$4p(x-h)=(y-k)^2$	$4p(y-k)=(x-h)^2$
Axis of Symmetry	$y=k$	$x=h$
Vertex	(h,k)	(h,k)
Focus	$(h+p,k)$	$(h,k+p)$
Directrix	$x=h-p$	$y=k-p$

Circles

Equation	$(x-h)^2+(y-k)^2=r^2$
Radius	r
Center	(h,k)

Ellipses

	Horizontal Major Axis	Vertical Major Axis
Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Center	(h,k)	(h,k)
Co-Vertices	$(h,\ k\pm b)$	$(h\pm b,\ k)$
Vertices	$(h\pm a,k)$	$(h,k\pm a)$
Major Axis	$y=k, \textit{length of } 2a$	$x=h, \textit{length of } 2a$
Minor Axis	$x=h, \textit{length of } 2b$	$y=k, \textit{length of } 2b$
Foci	$(h\pm c,k)$	$(h,k\pm c)$

a is the distance from center to vertices, b is the distance from center to co-vertices,
c is the distance from center to foci, $c^2=a^2-b^2$

Hyperbolas

	Horizontal Transverse Axis	Vertical Transverse Axis
Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Equations of Asymptotes	$y-k=\pm\frac{b}{a}(x-h)$	$y-k=\pm\frac{a}{b}(x-h)$
Center	(h,k)	(h,k)
Vertices	$(h\pm a,k)$	$(h,k\pm a)$
Transverse Axis	$y=k, \textit{length of } 2a$	$x=h, \textit{length of } 2a$
Conjugate Axis	$x=h, \textit{length of } 2b$	$y=k, \textit{length of } 2b$
Foci	$(h\pm c,k)$	$(h,k\pm c)$

a is the distance from center to vertices, b is the distance from center to co-vertices,
c is the distance from center to foci, $c^2=a^2+b^2$

Discriminant: B^2-4AC

Conic Tips
