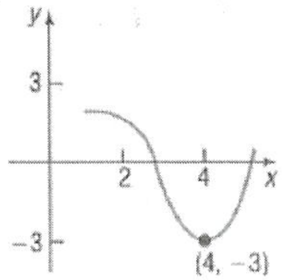
Name: Key

Unit J (limits) Review

For numbers 1 – 4, use the graph shown to determine if the limit exist. If it does, find its value.

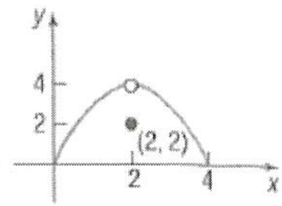
1. $\lim_{x\to 4} f(x)$

-3



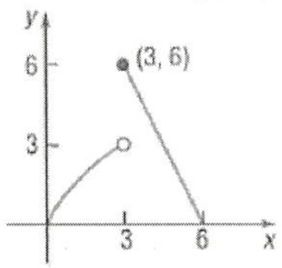
2. $\lim_{x\to 2} f(x)$

4



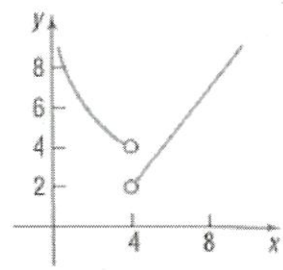
3. $\lim_{x\to 3} f(x)$

DNE

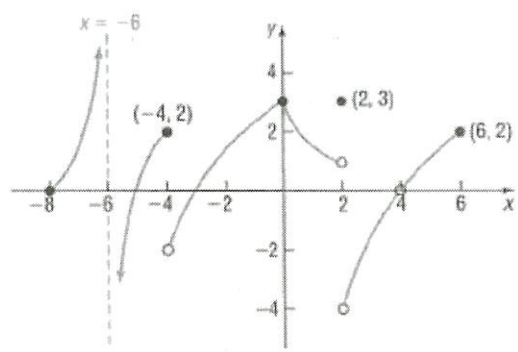


4. $\lim_{x\to 4} f(x)$

DNE



Use the following graph for numbers 5 – 10.



5. Find: $\lim_{x\to -6^-} f(x)$

∞

6. Find: $\lim_{x\to -6^+} f(x)$ $-\infty$

7. Find: $\lim_{x\to -4^-} f(x)$ 2

8. Find: $\lim_{x\to -4^+} f(x)$

-2

9. Find: $\lim_{x\to 2^-} f(x)$

1

10. Find: $\lim_{x\to 2^+} f(x)$

-4

For numbers 11 – 28, find each limit algebraically.

11. $\lim_{x\to 1} 5$

5

12. $\lim_{x\to 4} x$

4

13. $\lim_{x\to -1} \left(3x^2 - 5x\right)$

$3+5$

8

14. $\lim_{x\to 1} \sqrt{5x+4}$

3

15. $\lim_{x\to 0} \dfrac{x^2-4}{x^2+4}$

-1

16. $\lim_{x\to 2} \dfrac{x^2-4}{x^2-2x}$ $\dfrac{0}{0}$

$\dfrac{(x+2)\cancel{(x-2)}}{x\cancel{(x-2)}}$

$\dfrac{4}{2}$ 2

17. $\lim_{x\to -3}\frac{x^2-x-12}{x^2-9}$ $\frac{0}{0}$

$\frac{(x-4)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{-7}{-6} = \frac{7}{6}$

18. $\lim_{x\to -1}\frac{(x+1)^2}{x^2-1}$ $\frac{0}{0}$

$\frac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)}$ $\frac{0}{-2} = 0$

19. $\lim_{x\to 1}\frac{x^3-x^2+x-1}{x^4-x^3+2x-2}$ $\frac{0}{0}$

$\frac{x^2(x-1)+1(x-1)}{x^3(x-1)+2(x-1)} = \frac{(x^2+1)\cancel{(x-1)}}{(x^3+2)\cancel{(x-1)}}$

$\frac{2}{3}$

20. $\lim_{x\to 2}\frac{x^3-2x^2+4x-8}{x^2+x-6}$ $\frac{0}{0}$ $\boxed{\frac{8}{5}}$

$\frac{x^2(x-2)+4(x-2)}{(x+3)(x-2)} = \frac{(x^2+4)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

21. $\lim_{x\to -\infty}\frac{4x+8}{5x}$

$\frac{4}{5}$

22. $\lim_{x\to \infty}\frac{2x^2-4x}{x+1}$

∞

23. $\lim_{x\to \infty}\frac{5x+5}{7x^2+1}$

0

24. $\lim_{x\to -\infty} -5x^2-2x$

$-\infty$

25. $\lim_{x\to \infty} 3x^6+2x^3-4$

∞

26. $\lim_{x\to 4}\frac{\sqrt{x}+2}{x-4}$ $= \frac{4}{0}$

undefined, look at graph

DNE

27. $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ $\frac{0}{0}$ $\frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$

$\frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \boxed{\frac{1}{4}}$

28. $\lim_{x\to -\infty} \sin x$

DNE

For numbers 29 – 30, a piecewise function is given. Use properties of limits to find the indicated limit, or state that the limit does not exist.

29. $f(x)=\begin{cases} x-7 & \text{if } x<4 \\ 2x+3 & \text{if } x\ge 4 \end{cases}$

a) $\lim_{x\to 4^-} f(x)$

-3

b) $\lim_{x\to 4^+} f(x)$

11

c) $\lim_{x\to 4} f(x)$

DNE

30. $f(x)=\begin{cases} x^2-7 & \text{if } x<8 \\ x^3+2 & \text{if } x\ge 8 \end{cases}$

Find $\lim_{x\to 8} f(x)$

DNE

For numbers 31 – 32, find the slope of the tangent line to the graph of f at the given point <u>using the difference quotient.</u>

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

31. $f(x) = 3x + 5$ at $(1, 8)$

$\frac{3(h+1)+5-8}{h}$

$\frac{3\cancel{h}}{\cancel{h}}$

3

32. $f(x) = x^2 - 2x + 3$ at $(-1, 6)$

$\frac{(-1+h)^2-2(-1+h)+3-6}{h}$

$\frac{h^2-2h+1+2-2h+3-6}{h}$

$\frac{h^2-4h}{h}$

$\frac{\cancel{h}(h-4)}{\cancel{h}}$

-4

For numbers 33 –34, find the equation for the slope of the function at any point using the difference quotient $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Then use your answer to find the slope of the tangent line at $x = 4$.

33. $f(x) = -4x + 5$

$$\frac{-4(x+h)+5-[-4x+5]}{h}$$

$$\frac{-4x-4h+5+4x-5}{h}$$

$$\frac{-4\cancel{h}}{\cancel{h}} \quad \boxed{-4}, \quad -4$$

34. $f(x) = 2x^2 + 3x$,

$$\frac{2(x+h)^2+3(x+h)-[2x^2+3x]}{h}$$

$$\boxed{4x+3} \quad 19$$

$$\frac{2x^2+4xh+2h^2+3x+3h-2x^2-3x}{h}$$

$$\frac{2h^2+4xh+3h}{h} \qquad \frac{\cancel{h}(2h+4x+3)}{\cancel{h}}$$

For numbers 45 – 60, use the theorems to find $f'(x)$. Express answers without using fractional or negative exponents.

35. $f(x) = 12x^5$

$60x^4$

36. $f(x) = 3x^{-7}$

$-21x^{-8}$

37. $f(x) = x^3 - 2x + 1$

$3x^2-2$

38. $f(x) = 8$

0

39. $f(x)=\frac{10}{\sqrt{x}}-2x$

$10x^{-1/2}-2x$

$-5x^{-3/2}-2x$

$\frac{-5}{\sqrt{x^3}}-2$

40. $f(x) = 2x^{3/2} - 3x^{-1/3}$

$3x^{1/2}+x^{-4/3}$

$3\sqrt{x}+\frac{1}{\sqrt[3]{x^4}}$

41. $f(x)=2\sqrt[3]{x}+3$

$2x^{1/3}+3$

$\frac{2}{3}x^{-2/3}$

$\frac{2}{3\sqrt[3]{x^2}}$

42. $f(x)=x(3x^2-\sqrt{x})$

$3x^3-x^{3/2}$

$9x^2-\frac{3}{2}x^{1/2}$

$9x^2-\frac{3}{2}\sqrt{x}$

43. $f(x)=\frac{3}{x^3}-\frac{8}{x^2}+\frac{1}{x}$

$3x^{-3}-8x^{-2}+x^{-1}$

$-9x^{-4}+16x^{-3}-1x^{-2}$

$\frac{-9}{x^4}+\frac{16}{x^3}-\frac{1}{x^2}$

44. $f(x) = 3x^3 - 2\sqrt{x}$

$9x^2 - 2x^{1/2}$

$9x^2 - 1x^{-1/2}$

$9x^2 - \frac{1}{\sqrt{x}}$

45. $f(x) = x^9 - 3x^5 + 4x^2 + x$

$9x^8 - 15x^4 + 8x + 1$

46. $f(x) = \frac{-4}{x^3} - 2x^6 + 2\sqrt[3]{x^2}$

$-4x^{-3} - 2x^6 + 2x^{2/3}$

$12x^{-4} - 12x^5 + \frac{4}{3}x^{-1/3}$

$\frac{12}{x^4} - 12x^5 + \frac{4}{3\sqrt[3]{x}}$

47. $n(x) = (3x^2 - 2x)(x^3 + x^2)$

$(6x-2)(x^3+x^2)+(3x^2-2x)(3x^2-2x)$

$6x^4 + 6x^3 - 2x^3 - 2x^2 + 9x^4 - 6x^3 + 6x^3 - 4x^2$

$15x^4 + 4x^3 - 6x^2$

48. $r(x) = \frac{3x - 1}{x^2 + 2}$

$\frac{(3)(x^2+2)-(3x-1)(2x)}{(x^2+2)^2}$

$\frac{3x^2+6 - 6x^2+2x}{(x^2+2)^2}$

$\frac{-3x^2+2x+6}{(x^2+2)^2}$

49. $h(x) = (-4 + 2x^2)(2x + 3)$

$(4x)(2x+3) + 2(-4+2x^2)$

$8x^2 + 12x - 8 + 4x^2$

$12x^2 + 12x - 8$

50. $k(x) = \frac{3x^3 + 4}{2x^2 - 1}$

$\frac{(9x^2)(2x^2-1) - (3x^3+4)(4x)}{(2x^2-1)^2}$

$\frac{18x^4 - 9x^2 - 12x^4 - 16x}{(2x^2-1)^2}$

$\frac{6x^4 - 9x^2 - 16x}{(2x^2-1)^2}$