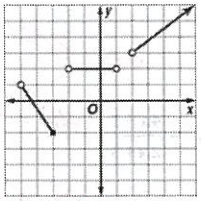
Pre – Calculus
REVIEW for Midterm

Name: Key

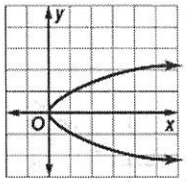
1

1. Which relation is a function?

A



B



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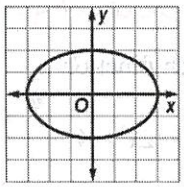
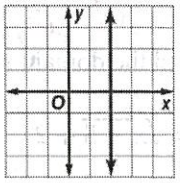


D



A

vertical line test!

2. Describe the end behavior of the graph of $f(x) = -2x^3 + 3x - 4$ using limit notation.

$\lim_{x \to \infty} f(x) = -\infty$ $\lim_{x \to -\infty} f(x) = \infty$

A

3. Given $f(x) = \begin{cases} -2\,|x+7| \text{ if } x < -1 \\ -4 \text{ if } x \geq -1 \end{cases}$

a. Find $f(-1)$

-4

b. Find $f(-10)$ ← Typo

$-2|-10+7| = -2|-3| = 2(3) = 6$

A

4. Which function has infinite discontinuity?

D

A $f(x) = \frac{x^2-36}{x-6}$ $(x-6)(x+6)$ hole!

B $f(x) = x^5 - x^3$ C

C $f(x) = \begin{cases} 2x \text{ if } x < 0 \\ 3x+1 \text{ if } x \geq 0 \end{cases}$ J

D $f(x) = \frac{1}{3x-5}$

A

5. Given the parent function $p(x) = x^3$, what translation occurs in the graph of $p(x) = -(x+4)^3 - 3$?

L4, D3, x-axis reflect

A

6. If $f(x) = 2x - 3$ and $g(x) = x^2 - 7$, find

a) $f(g(x))$

$2(x^2-7) - 3$

$2x^2 - 14 - 3$

$2x^2 - 17$

b) find $g(f(x))$

$(2x-3)^2 - 7$

$4x^2 - 12x + 9 - 7$

$4x^2 - 12x + 2$

A

7. A toy rocket is launched from the ground straight upward. The height of the rocket above the ground, in feet, is modeled by $h(t) = -16t^2 + 64t$, where t is the time in seconds. What is the relative maximum of the function?

Max in calc OR find vertex

$-16(t-4) = h(t)$ 64

A

8. Determine the equation for the inverse of the given functions.

a) $f(x) = 3x^2 - 4$

$x = 3y^2 - 4$

$\sqrt{\frac{x+4}{3}} = \sqrt{\frac{3y^2}{3}}$

$y = \sqrt{\frac{x+4}{3}}$

b) $g(x) = \frac{1}{2}\sqrt{x+4}$

$x = \frac{1}{2}\sqrt{y+4}$

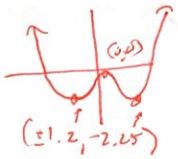
$(2x) = (\sqrt{y+4})^2$

$4x^2 = y + 4$

$y = 4x^2 - 4$

A

9. Describe where the function $f(x) = x^4 - 3x^2$ is increasing and/or decreasing. (x-values!) A



Increasing: $(-1.2, 0) \cup (1.2, \infty)$

Decreasing: $(-\infty, -1.2) \cup (0, 1.2)$

10. Find the domain and range of each function.

a) $y = \sqrt{x-5} + 2$

D: $[5, \infty)$

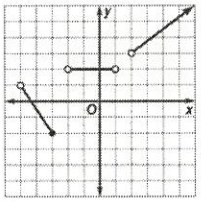
R: $[2, \infty)$

b) $y = 2x^3 - 4$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

c)



D: $(-5, 3] \cup (-2, 1) \cup (2, \infty)$

R: $[-2, 1) \cup [2] \cup (3, \infty)$

d) $y = \frac{4x+1}{2x+3}$ A

VA: $x = -\frac{3}{2}$

HA: $y = 2$

R: $(-\infty, 2) \cup (2, \infty)$

D: $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

11. Determine the vertical and horizontal asymptote equations for each function. A

a) $f(x) = \frac{x^2+4}{x^2-1}$

VA: $x = \pm 1$

HA: $y = 1$

b) $f(x) = \frac{x^2}{3x+4}$

VA: $x = -\frac{4}{3}$

HA: none

c) $f(x) = \frac{x-1}{x^2-4}$

VA: $x = \pm 2$

HA: $y = 0$

12. For $f(x) = \frac{x-1}{x^2-4}$, find $\lim_{x \to \infty} f(x)$ $= 0$ A

For 13 – 16, evaluate. Round to the nearest hundredth if necessary. B

13. log15

1.18

14. $\log_6 61$

2.29

15. $\log_5 \frac{1}{125}$

$\log_5 5^{-3}$

-3

16. $\ln e^3$

3

17. Solve for x.

a) $\ln(x+4) + \ln x = \ln 5$

$\ln x(x+4) = \ln 5$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5, 1$

b) $\log_2 2x = 7$

$2x = 2^7$

$x = 2^6$

c) $\log(x+3) + \log x = 1$ Typo B

$\log x(x+3) = 1$

$x^2 + 3x = 10^1$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5, 2$

18. Expand $\log_3 x^5 y^4 \sqrt[3]{z}$ B

$\log_3 x^5 + \log_3 y^4 + \log_3 \sqrt[3]{z}$

$5\log_3 x + 4\log_3 y + \frac{1}{3}\log_3 z$

19. Condense $\frac{1}{4}\ln x - \ln(y+4)$ B

$\ln \sqrt[4]{x}$

$(y+4)$

neat:

$\ln \frac{\sqrt[4]{x}}{y+4}$

20. Solve for x. Round to the nearest hundredth if necessary.

a) $2^x = 14$

b) $3^{2x-1} = 17$

21. Find the domain and range of the function $y = \frac{1}{2}(3)^x$.

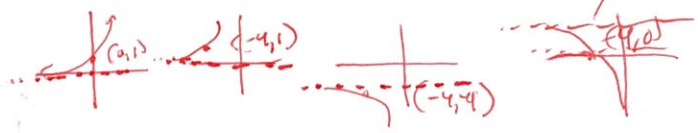
D: $(-\infty, \infty)$

R: $(0, \infty)$

For numbers 22 – 24, use the following function $f(x) = -4e^{x+4} + 4$

22. What transformations have been performed to the graph of $y = e^x$?

23. State the domain, range, asymptote equations, and end behavior for the function $f(x)$



D: $(-\infty, \infty)$

R: $(-\infty, 4)$

Asym: $y = 4$ (HA)

VA: (none)

End B

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = 4$

24. Determine the intervals when the function $f(x)$ is increasing and decreasing.

Decreasing: $(-\infty, \infty)$

Increasing: none

25. There are initially 500 rabbits in a population in 2017. The rabbit population is growing at an exponential rate of 4.2% per year. Determine the year in which the population of rabbits will triple.

$500(1+.042)^t = 1500$

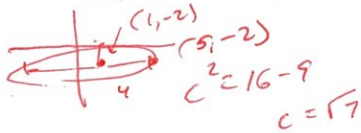
$1.042^t = 3$

$t = \log_{1.042} 3$

$t = 26.7$

During 2043 so: 2044

For numbers 26 – 28, refer to the ellipse represented by $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$.



$a = 4$ $b = 3$ $c^2 = 16 - 9$ $c = \sqrt{7}$

26. Find the coordinates of the center.

a) (1, 2) b) (–1, –2) c) (–1, 2) d) (–2, 1) e) (1, –2)

27. Find the coordinates of the foci.

a) $(1 \pm \sqrt{7}, -2)$ b) (5, –2) , (–3, –2) c) $(1, -2 \pm \sqrt{7})$ d) (1, 4), (1, –8) e) (–4, –2) , (6, –2)

28. Find the coordinates of the vertices and co-vertices.

a) (1, 2), (1, –6), (4, –2), (–2, –2)

b) (5, –2), (–3, –2), (1, 1), (1, –5)

c) (4, 2), (–2, 2), (1, 1), (1, –5)

d) (5, –2), (–3, –2), (1, 2), (1, –6)

e) Ellipses don't have co-vertices

29. Find the coordinates of the foci for the hyperbola $\frac{y^2}{4}-\frac{x^2}{2}=1$.

a) $(0,\pm\sqrt{2})$ b) $(0,\pm\sqrt{6})$ c) $(\pm\sqrt{2},0)$ d) $(\pm\sqrt{6},0)$ e) None of these

30. Write the standard form of the equation of the hyperbola for which the transverse axis is 4 units long and vertical and the conjugate axis is 3 units long.

a) $\frac{(x-1)^2}{2.25}-\frac{(y+4)^2}{4}=1$

b) $\frac{(y+4)^2}{2.25}-\frac{(x-1)^2}{4}=1$

c) $\frac{(y+4)^2}{2.25}+\frac{(x-1)^2}{4}=1$

d) $\frac{(x-1)^2}{4}-\frac{(y+4)^2}{2.25}=1$

e) $\frac{(y+4)^2}{4}-\frac{(x-1)^2}{2.25}=1$

31. What is the directrix of the parabola with equation $x^2 = -28y$?

a) $x = 7$ b) $x = 28$ c) $y = -7$ d) $y = 7$ e) $y = 28$

32. Determine the orientation of the parabola: focus (0, 4), directrix $y = 1$

a) up b) down c) left d) right e) none

33. Which of the following is an arithmetic sequence?

a) 2, 4, 8, 14, 22, … b) 1, 5, 6, 10, 11, … c) 3, 9, 21, 39, 63, … d) –3, 0, 6, 15, 27, … e) 3, 8, 13, 18, 23, …

34. What is a rule for the nth term of the arithmetic sequence with $a_8 = 17$ and $a_{14} = 3$?

35. What is the common ratio of an infinite geometric series whose sum is 125 and the first term is $a_1 = 625$?

36. Find the sum of the series: $\sum_{k=1}^{11}(1-3k)$

a) –396 b) –374 c) –198 d) –187 e) –100

37. What is the sum of the series: $3 + 1.8 + 1.08 + 0.648 + \ldots$?

a) 5 b) 7.5 c) 8 d) 10 e) does not converge

$r = .6$ or $\frac{3}{5}$ $\quad S_n = \frac{3}{1-\frac{3}{5}}$

38. What is S_{25} for the arithmetic series $4 + 4.2 + 4.4 + 4.6 + 4.8 + \ldots$?

$a_n = 4 + (n-1).2$

$a_n = .2n + 3.8$

$a_{25} = .2(25) + 3.8$

$S_{25} = \frac{25}{2}(4 + 8.8)$ 160

39. What is the twelfth term of the sequence of a geometric sequence; -6, 18, -54…..

$a_{12} = -6(-3)^{11}$ 1062882

40. How do you write the series $4 + 6 + 8 + 10$ using sigma notation?

a) $\sum_{k=1}^{4} 2k$ b) $\sum_{k=1}^{4} 2k+2$ c) $\sum_{k=1}^{4} k+1$ d) $\sum_{k=1}^{4} k+3$ e) $\sum_{k=1}^{4} 2k-2$

$a_n = 4 + (n-1)2$

$a_n = 2n + 2$

41. How do you write the series $4 + 6 + 9 + 13.5$ using sigma notation?

a) $\sum_{n=1}^{4} 1.5(n)^4$ b) $\sum_{n=1}^{4} 1.5(4)^{n-1}$ c) $\sum_{n=1}^{4} 4(1.5)^{n-1}$ d) $\sum_{n=1}^{4} n(1.5)^4$ e) $\sum_{n=1}^{4} 1.5(n-1)^4$

$r = \frac{3}{2}$ $\quad a_n = 4\left(\frac{3}{2}\right)^{n-1}$

42. Which series is represented by $\sum_{k=2}^{4} (2k^2 + k)$?

a) $3 + 10 + 21$ b) $10 + 21 + 36$ c) $3 + 7 + 11$ d) $10 + 14 + 18$ e) $10 + 21 + 32$

$[2(2)^2 + 2] + [2(3)^2 + (3)] + [2(4)^2 + (4)]$

$10 + 21 + 36$

43. What is the common ratio of the sequence $\frac{3}{100}, \frac{3}{50}, \frac{3}{25}, \frac{3}{12.5}, \ldots$?

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) 2 d) 4 e) 6

44. What is S_6 for the geometric series $0.25 - 0.75 + 2.25 - 6.75 + \ldots$?

D

$a_6 = .25(-3)^5$ $\quad S_6 = .25\,\frac{1-(-3)^6}{1-(-3)}$ $\quad -45.5$

45. What is the common difference of the sequence 3, 4.5, 6, 7.5, …?

D

$d = 1.5$

46. An infinite geometric series has a sum of 200 and a common ratio $\frac{4}{5}$. Which is the first term of this series?

D

$200 = \frac{a_1}{1-(\frac{4}{5})}$ $\quad 40$

47. Find AB if $A = \begin{bmatrix} -1 & 3 \\ 0.5 & -0.2 \end{bmatrix}$ and $B = \begin{bmatrix} -0.4 & 1.2 \\ 5 & -0.1 \end{bmatrix}$.

E

a) $\begin{bmatrix} 0.62 & -1.2 \\ -1.5 & 15.4 \end{bmatrix}$ b) $\begin{bmatrix} -0.62 & 1.2 \\ 1.5 & -15.4 \end{bmatrix}$ c) $\begin{bmatrix} 15.4 & -1.5 \\ -1.2 & 0.62 \end{bmatrix}$ d) $\begin{bmatrix} -15.4 & 1.5 \\ 1.2 & -0.62 \end{bmatrix}$ e) not possible

48. Solve the following system of equations using an inverse matrix.

$-x + 2y - 3z = 11 \quad 2x + z = 4 \quad x - y + 2z = -5$

E

a) (–3, –4, 2) b) (3, 4, –2) c) (3, –4, 2) d) (–3, 4, –2)

49. The table shows several boxes of assorted candy available at a candy shop. What is the price per pound for each candy?

E

Box	Chocolate	Taffy	Nougat	Price ($)
Grand Edition	10	5	0	12.25
Special Edition	10	5	5	16.25
Deluxe Edition	15	10	5	24.25

a) ($0.85, $0.75, $0.80) b) ($0.75, $0.80, $0.85) c) ($0.80, $0.75, $0.85) d) ($0.75, $0.85, $0.80)