Pre-Calculus Name: Key (1)

Final Review Unit

For numbers 1 – 3, find the exact value

S | A
T | C

1. $\tan\frac{3\pi}{4}$ (-1)

2. $\cos-\frac{\pi}{3}$ or $\cos\frac{5\pi}{3}$ (+1/2)

3. $\csc\frac{5\pi}{6}$ find $\sin\frac{5\pi}{6}=\frac{1}{2}$ then $\frac{2}{1}$ (2)

4. a) convert $\frac{11\pi}{6}$ radians to degrees. $\frac{11\pi}{6}\left(\frac{180^\circ}{\pi}\right)$ 330° or on U.C.

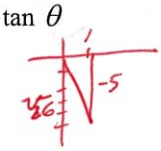
b) convert 315° to radians. $315^\circ\left(\frac{\pi}{180^\circ}\right)$ $\frac{7\pi}{4}$

5. If the terminal ray of θ passes through the given point, find csc θ, sec θ, and tan θ

a) (-2, -4) 

$\csc\theta=\frac{H}{O}=\frac{\sqrt{20}}{-4}=\frac{2\sqrt{5}}{-4}=-\frac{\sqrt{5}}{2}$

$\sec\theta=\frac{H}{A}=\frac{2\sqrt{5}}{-2}=-\sqrt{5}$ $\tan\theta=2$

b) (1, –5) 

$\frac{H}{O}=\frac{\sqrt{26}}{-5}$ $\frac{H}{A}=\sqrt{26}$ $\frac{O}{A}=-5$

6. A circular sector has a radius of 4 and a central angle of 150°. Find the arc length using $s=r\theta$.

$s=r\theta=4\left(\frac{5\pi}{6}\right)=\frac{10\pi}{3}$ $150^\circ\left(\frac{\pi}{180^\circ}\right)=\frac{5\pi}{6}$ must be radians!

7. What is the exact value of the sin 135°? $-\frac{\sqrt{2}}{2}$

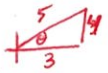
8. Identify the phase shift (horizontal translation) of $y=2\sin(3x+\pi)-5$

$y=2\sin(3)\left(x+\frac{\pi}{3}\right)-5$ Left $\frac{\pi}{3}$

9. Identify the amplitude and period of $y=-4\cos(2x+\frac{\pi}{3})$ (abs!)

$a=4$ $P=\pi$ $-4\cos(2)\left(x+\frac{\pi}{6}\right)$ $P=\frac{2\pi}{b}=\frac{2\pi}{2}=\pi$

10. Find the exact value in radians.



arc sin → Q1, Q4
arc cos → Q1, Q2
arc tan → Q1, Q4

a) $\arctan-\frac{\sqrt{3}}{3}$ $\frac{5\pi}{6}$

b) $\arccos-\frac{1}{2}$ $\frac{2\pi}{3}$

c) $\csc\left(\cos^{-1}\frac{\sqrt{2}}{2}\right)$ $\csc\left(\frac{\pi}{4}\right)$ $\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$ $\sqrt{2}$

d) $\sec\left(\tan^{-1}\frac{3}{4}\right)$ $\frac{H}{A}$ $\frac{5}{3}$

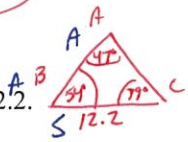
11. An airplane is at an elevation of 45,000 ft and approaches the airport with an angle of descent of 15°. What is the distance between the airport and the point on the ground directly below the plane? What is the distance the plane needs to travel to get to the runway?

$\tan75^\circ=\frac{x}{45{,}000}$ $x=167{,}942$ ft

12. An avid bird watcher spots a bird in a tree at an angle of elevation of 32°. If the bird watcher is 26 feet from the bottom of the tree, how far off the ground is the bird?

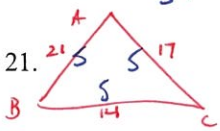
$\tan32^\circ=\frac{x}{26}$ 16.25 ft

13. In ΔABC, find the measure of c given that $m\angle A=47^\circ$, $m\angle B=54^\circ$, and $a=12.2$.



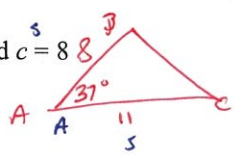
$\frac{\sin54^\circ}{c}=\frac{\sin47^\circ}{12.2}$ $c=13.5$

14. In ΔABC, find the measure of $\angle B$ given that $a=14$, $b=17$, and $c=21$.



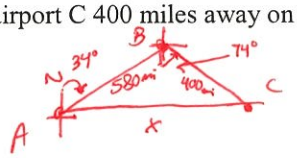
$17^2=14^2+21^2-2(14)(21)\cos\beta$ $\angle B=54^\circ$

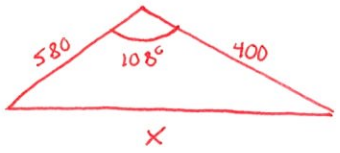
15. In ΔABC, find the measure of a given that $m\angle A=37^\circ$, $b=11$ and $c=8$



$a^2=11^2+8^2-2(11)(8)\cos37^\circ$ $a=6.67$

(F) 16. A plane leaves airport A and travels 580 miles to airport B on a bearing of N34°E. The plane later leaves airport B and travels to airport C 400 miles away on a bearing of S74°E. Find the distance from airport A to airport C to the nearest tenth of a mile.



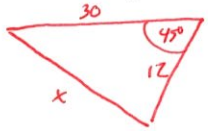


$x^2 = 580^2 + 400^2 - 2(580)(400)\cos 108°$

$x = 800$ mi

799.9 mi

(F) 17. You are on a fishing boat that leaves its pier and heads east. After traveling 30 miles, there is a report warning of rough seas directly south. The captain turns the boat and follows a bearing S45°W for 12 miles.
At this time, how far are you from the boat's pier? Round to the nearest tenth of a mile.



$x^2 = 12^2 + 30^2 - 2(12)(30)\cos 45°$

$x = 23.1$ mi

For numbers 18 - 22, match the trigonometric function to the correct graph.

(F)(E) 18. $y = -3\sin\left(x + \frac{\pi}{4}\right)$ x-axis reflect; R $\frac{\pi}{4}$; Amp 3 (V.S.)

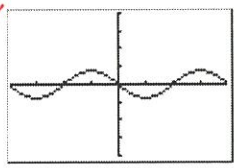
(F)(D) 19. $y = 5\cos 2x$ amp: 5 (V.S.); H.C.

(F)(A) 20. $y = \frac{3}{4}\sin(x - \pi)$ amp: $\frac{3}{4}$ (V.C.); R: π

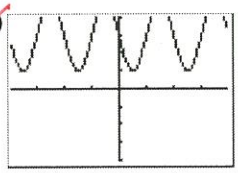
(F)(C) 21. $y = \cos 2(x + \pi) - 1$ H.C.; L π; D 1

(F)(B) 22. $y = -2\sin 2x + 3$ x-axis reflect; amp 2 (V.S.); Up 3

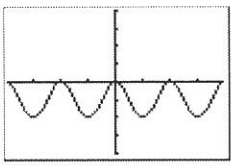
a)



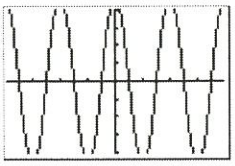
b)



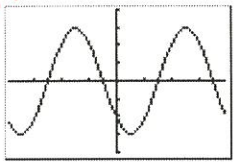
c)



d)



e)



For numbers 23 – 26, match the appropriate trigonometric expression with one of the following. (You may only use each answer choice once.)

(G) 23. $(1 - \sin^2 x)(\sec x)$ $= \cos^2 x \cdot \frac{1}{\cos x} = \cos x$ (D)

(G) 24. $\frac{\csc^2 x - 1}{\cos^2 x} = \frac{\cot^2 x}{\cos^2 x} = \frac{\frac{\cos^2 x}{\sin^2 x}}{\cos^2 x} = \frac{1}{\sin^2 x} = \csc^2 x$ (E)

(G) 25. $\csc x \sin x$ (1) (A)

(G) 26. $\tan^2 x - \sec^2 x$ $(\sec^2 - 1) - \sec^2 x$ -1 (B)

a) 1

b) -1

c) $\csc x$

d) $\cos x$

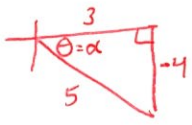
e) $\csc^2 x$

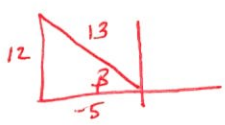
(G) 27. Which of the following is equal to tan 15°? $\tan(45° - 30°)$

a) $\frac{1 - \tan 45° \tan 30°}{\tan 45° + \tan 30°}$ b) $\frac{1 + \tan 45° \tan 30°}{\tan 45° \tan 30°}$ c) $\frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$ d) $\frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$ e) none of these

(G) 28. Which is $\tan(\alpha + \beta)$ if $\sin\alpha = -\frac{4}{5}$ with $270° < \alpha < 360°$ (Q4) and if $\cos\beta = -\frac{5}{13}$ with $90° < \beta < 180°$ (Q2)?

a) $-\frac{56}{15}$ b) $-\frac{56}{33}$ c) $-\frac{33}{56}$ d) $\frac{56}{33}$ e) $\frac{56}{15}$





$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \frac{-\frac{4}{3} + \frac{12}{-5}}{1 - \left(-\frac{4}{3}\right)\left(\frac{12}{-5}\right)} = \frac{\frac{-56}{15}}{\frac{-11}{5}} = \frac{56}{33}$

(G) 29. Use a sum or difference identity to find the exact value of $\sin 15°$.

a) $\frac{-\sqrt{2}-\sqrt{6}}{4}$ b) $\frac{\sqrt{6}+\sqrt{2}}{4}$ c) $\frac{\sqrt{6}-\sqrt{2}}{4}$ d) $\frac{\sqrt{2}-\sqrt{6}}{4}$ e) $\frac{\sqrt{6}+\sqrt{2}}{2}$

(G) 30. If $\sec\theta = -\frac{5}{3}$ and θ has its terminal side in Quadrant II, find the exact value of $\cos 2\theta$.

a) $-\frac{24}{7}$ b) $-\frac{7}{25}$ c) $\frac{7}{25}$ d) $\frac{24}{25}$ e) $\frac{24}{7}$

(G) 31. Use a half-angle identity to find the exact value of $\sin 105°$.

a) $-\frac{1}{2}\sqrt{2+\sqrt{3}}$ b) $-\frac{1}{2}\sqrt{1+\sqrt{3}}$ c) $-\frac{1}{2}\sqrt{2-\sqrt{3}}$ d) $\frac{1}{2}\sqrt{2+\sqrt{2}}$ e) $\frac{1}{2}\sqrt{2+\sqrt{3}}$

(G) 32. What is a solution of the equation $4\cos x + 2 = 0$?

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{4}$ e) $\frac{5\pi}{3}$

(G) 33. One of the solutions for the equation $3\cos 2x + \sin x + 2 = 0$, for $0 \le x < 2\pi$, is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{4\pi}{3}$ e) none of these

(G) 34. What is a solution of the equation $\tan\frac{x}{2} = 1$?

a) $\frac{3\pi}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{5\pi}{2}$ e) $\frac{5\pi}{4}$

(G) 35. Solve $\cos^2 x + \sin x + 1 = 0$ for $0 \le \theta < 2\pi$.

a) $0, \pi$ b) $\frac{3\pi}{2}$ c) π d) $\frac{\pi}{2}, \frac{3\pi}{2}$ e) $\frac{\pi}{2}$

(H) 36. Given the polar coordinate $\left(4, \frac{5\pi}{4}\right)$, which of the following is the rectangular coordinate?

a) $\left(-2\sqrt{2}, -2\sqrt{2}\right)$ b) $\left(2, 2\sqrt{3}\right)$ c) $\left(2\sqrt{2}, 2\sqrt{2}\right)$ d) $\left(-2\sqrt{3}, -2\right)$ e) $\left(-2\sqrt{2}, 2\sqrt{3}\right)$

(H) 37. Given the polar coordinate $\left(-2, \frac{5\pi}{6}\right)$, which of the following is the rectangular coordinate?

a) $\left(-\sqrt{3}, -1\right)$ b) $\left(-2\sqrt{3}, 2\right)$ c) $\left(\sqrt{3}, -1\right)$ d) $\left(2\sqrt{3}, -2\right)$ e) $\left(-\sqrt{3}, 1\right)$

(H) 38. Given the rectangular coordinate $(2, -2)$, which of the following is the polar coordinate?

a) $\left(2, \frac{\pi}{4}\right)$ b) $\left(\sqrt{2}, \frac{7\pi}{4}\right)$ c) $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ d) $\left(2\sqrt{2}, \frac{7\pi}{4}\right)$ e) $\left(\sqrt{2}, \frac{\pi}{4}\right)$

H 39. Given the rectangular coordinate $(-2, 2\sqrt{3})$, which of the following is the polar coordinate?

a) $\left(4, \frac{\pi}{3}\right)$ b) $\left(4, \frac{5\pi}{6}\right)$ c) $\left(2, \frac{2\pi}{3}\right)$ d) $\left(2, \frac{\pi}{3}\right)$ e) $\left(4, \frac{2\pi}{3}\right)$

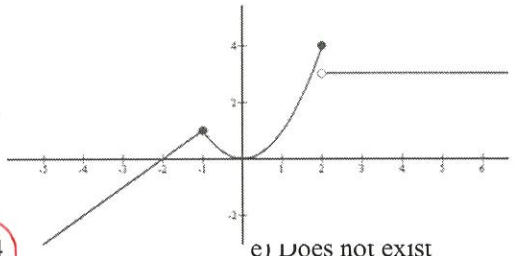
H 40. Given the rectangular equation $x^2 + y^2 = 16$, which of the following is the polar equation?

a) $r = 4$ b) $r = 256$ c) $r = \pm 4$ d) $r = \pm 256$ e) $r = 16$

H 41. Given the polar equation $r = 9 \csc \theta$, which of the following is the rectangular equation?

a) $y = 9$ b) $x + y = 9$ c) $y = \frac{1}{9}$ d) $x = \frac{1}{9}$ e) $x = 9$

For numbers 42 – 44, use the graph of the piecewise function to find each value.



J 42. $\lim_{x \to 2^-} f(x)$

a) 1 b) 2 c) 3 d) 4 e) Does not exist

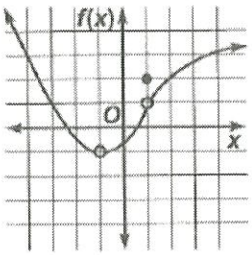
J 43. $\lim_{x \to 2^+} f(x)$

a) 1 b) 2 c) 3 d) 4 e) Does not exist

J 44. $\lim_{x \to 2} f(x)$

a) 1 b) 2 c) 3 d) 4 e) Does not exist

For numbers 45 & 46 use the graph of $y = f(x)$ to find each value.



J 45. $\lim_{x \to -1} f(x)$

a) –1 b) 0 c) 1 d) 2 e) 3

J 46. $\lim_{x \to 1} f(x)$

a) –1 b) 0 c) 1 d) 2 e) 3

J 47. Evaluate: $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

a) 0 b) 1 c) 3 d) 6 e) 9

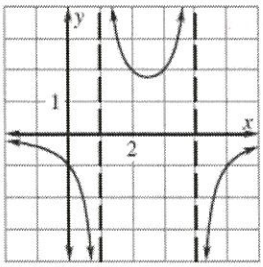
48. Evaluate: $\lim_{x\to 3}\frac{x+7}{3-x}$ $\frac{10}{0}$ DNE

a) 0 b) 1 c) 2 d) 3 e) Does not exist

49. Evaluate: $\lim_{x\to\infty}\frac{9x-6}{3x+2}$

a) $-\infty$ b) 1 c) 3 d) ∞ e) Does not exist

For numbers 50 & 51 use the graph of $y = f(x)$ to find each value.



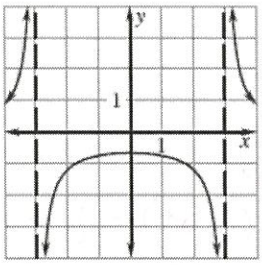
50. $\lim_{x\to\infty} f(x)$

a) –1 b) –0.5 c) 0 d) 0.5 e) Does not exist

51. $\lim_{x\to-\infty} f(x)$

a) –1 b) –0.5 c) 0 d) 0.5 e) Does not exist

52. Find: $\lim_{x\to 3} f(x)$



a) –1 b) 0 c) 1 d) 2 e) Does not exist

For numbers 53 – 54, use the following information: Given: $f(x)=\begin{cases}4x+3 \text{ if } x<4\\ 7x-8 \text{ if } x\geq 4\end{cases}$

53. Find: $\lim_{x\to 4^+} f(x)$

a) 16 b) 19 c) 20 d) 28 e) Does not exist

54. Find: $\lim_{x\to 4} f(x)$

a) 16 b) 19 c) 20 d) 28 e) Does not exist

For 55-58, find the derivative of each function.

55. $f(x) = 3x^2 + x$ $6x+1$

a) $x^3 + \frac{x^2}{2}$ b) $6x$ c) $6x + 1$ d) $x^3 - x^2$

56. $f(x) = 3(x-2)^2 + 5$ $\quad 3(x^2-4x+4)+5$

a) $6x - 12$ b) $x - 2$ c) $6(x-2)+5$ d) $3x^2 - 12x + 17$

$3x^2-12x+17$

$6x-12$

57. $f(x) = \frac{3}{x^2} - \frac{2}{x} + 4$ $\quad 3x^{-2} - 2x^{-1} + 4$

$-6x^{-3} + 2x^{-2}$

a) $\frac{6}{x^3} - \frac{2}{x^2}$ b) $-\frac{6}{x^3} - \frac{2}{x^2}$ c) $-\frac{6}{x^3} + \frac{2}{x^2}$ d) $\frac{6}{x^3} + \frac{2}{x^2}$ e) $-6x^3 + 2x^2$

$\frac{-6}{x^3} + \frac{2}{x^2}$

58. $f(x) = -2x^3 - x^2$ $\quad -6x^2 - 2x$

a) $-\frac{x^4}{2} - \frac{x^3}{3}$ b) $-\frac{x^4}{2} - x^2$ c) $-6x^2 - x^2$ d) $-6x^2 - 2x$

For 59 -60, use the Quotient Rule to find the derivative of each function.

59. $h(x) = \frac{3-2x}{3+2x}$

a) $h'(x) = \frac{-12}{3+2x}$ b) $h'(x) = \frac{-12}{(3+2x)^2}$ c) $h'(x) = \frac{-12-8x}{(3+2x)^2}$ d) $h'(x) = \frac{-12}{(3-2x)^2}$

60. $g(x) = \frac{x^2+4}{3-x^2}$

a) $g'(x) = \frac{-2x}{(3-x^2)^2}$ b) $g'(x) = \frac{14x}{(3-x^2)}$ c) $g'(x) = \frac{14x-4x^3}{(3-x^2)^2}$ d) $g'(x) = \frac{14x}{(3-x^2)^2}$

$$\frac{-2(3+2x) - [2(3-2x)]}{(3+2x)^2}$$

$$\frac{-6-4x-6+4x}{(3+2x)^2}$$

$$\frac{-12}{(3+2x)^2}$$

$$\frac{2x(3-x^2) - [(-2x)(x^2+4)]}{(3-x^2)^2}$$

$$\frac{6x-2x^3+2x^3+8x}{(3-x^2)^2}$$

$$\frac{14x}{(3-x^2)^2}$$