

## Unit 4: Exponents

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**EQ 1:** How are the properties of exponents applied?

**EQ 2:** What real life problems can be modeled by exponents?

**\*\*\*=quiz to follow**

## 7.1 Properties of Exponents

Simplify each of the following completely.

1)  $(10ab)^0$

2)  $10ab^0$

3)  $5x^{-3}$

4)  $(5x)^{-3}$

5)  $a^6 \cdot a^{10}$

6)  $y^{-5} \cdot y^2 \cdot y$

7)  $(x^5)^6$

8)  $(2^2)^{-2}$

9)  $(5x^2y^3)^3$

10)  $(2a^{-4}b^3c^2)^{-2}$

11)  $\frac{x^4}{x^2}$

12)  $\frac{a^5b^8c^4}{a^2b^7c^2}$

$$13) \left( \frac{x^4}{y^2} \right)^5$$

$$14) \left( \frac{5a^2b^3}{c^2} \right)^3$$

$$15) \frac{2w^2x^{-3}y^{-8}z^0}{20a^{-4}b^{-2}c^5d^2}$$

$$16) \frac{a^4b^2c^{10}}{a^6b^4c^{12}}$$

$$17) (-5ab)(2a^2b^2)(10a^{-1}b)$$

$$18) (3a^2b)^{-2}(2a^4b^2)^2$$

$$19) \left( \frac{2x^4y^4}{3x^2y^2} \right)^{-2}$$

$$20) \left( \frac{a^4b^5c^{10}}{10} \right)^{-1}$$

$$21) (-3a^2b^5)^0(-7a^{10}b^{10})^0(2ab)^0$$

$$22) \left( \frac{(10xy^2)(2x^2y^{-4})}{1} \right)^0$$

## Exponent properties summarized

### 1. Zero Exponents

$$a^0 = 1$$

$$\text{examples: } x^0 = 1 \quad 100^0 = 1 \quad 5xyz^0 = 5xy \quad (5xyz)^0 = 1$$

### 2. Negative Exponents

$$x^{-n} = \frac{1}{x^n}$$

$$\text{examples: } 10^{-2} = \frac{1}{10^2} = \frac{1}{100} \quad \frac{5x^{-1}y^5z^{-5}}{2a^{10}b^{-4}c^2} = \frac{5y^5b^4}{2a^{10}c^2x^1z^5}$$

### 3. Multiplying Powers with the Same Base

$$a^m \cdot a^n = a^{m+n}$$

$$\text{examples: } 2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32 \quad x^{10} \cdot x^5 = x^{10+5} = x^{15}$$

### 4. Raising a Power to a Power

$$(a^m)^n = a^{mn}$$

$$\text{examples: } (a^5)^2 = a^{5(2)} = a^{10} \quad (2^2)^{-3} = 2^{2(-3)} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

### 5. Raising a Product to a Power

$$(ab)^n = a^n \cdot b^n$$

$$\text{examples: } (2x^2)^3 = 2^3x^6 = 8x^6 \quad (2x^2)^{-3} = 2^{-3}x^{-6} = \frac{1}{2^3x^6} = \frac{1}{8x^6}$$

### 6. Dividing Powers with the Same Base

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\text{example: } \frac{5^3}{5^5} = 5^{3-5} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

### 7. Raising a Quotient to a Power

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{examples: } \left(\frac{x^5}{y^2}\right)^3 = \frac{x^{15}}{y^6} \quad \left(\frac{2^3}{4}\right)^2 = \frac{2^6}{4^2} = \frac{64}{16} = 4$$

### Quiz Review Sections 7.1 & 7.2

For numbers 1 – 9, evaluate the expression.

1.  $x^8 \cdot x^3$

2.  $(y^4)^2$

3.  $(x \cdot y)^2$

4.  $(2a)^5$

5.  $(5b)^2 \cdot b$

6.  $2y^4 \cdot 4y^2 \cdot 3y^{12}$

7.  $(-2x)^2$

8.  $-(3y^5)^2$

9.  $(-6^2)^3$

In numbers 10 - 15, evaluate the exponential expression. Write your answer as a fraction in simplest form.

10.  $3^{-4}$

11.  $\left(\frac{2}{5}\right)^{-1}$

12.  $\left(\frac{1}{5}\right)^{-3}$

13.  $(-100)^0 \cdot \frac{1}{6^{-2}}$

14.  $(2^{-3})^{-2}$

15.  $5^4 \cdot 5^{-4}$

In numbers 16 -18, rewrite the expression with positive exponents.

16.  $x^{-4}$

17.  $y^{-4}z^5$

18.  $(7xy)^{-3}$

For numbers 19-21, simplify and then evaluate if  $x = 1$  and  $y = 2$ .

19.  $x^2y^2$

20.  $(3x)^2y$

21.  $(xy^2)^3$

# Exponent reminder

- When multiplying: add the exponents

$$\text{Ex: } a^5 \cdot a^9 = a^{5+9} = a^{14}$$

$$\text{Ex: } 6^3 \cdot 6^4 = 6^{3+4} = 6^7$$

- When raising an exponent to an exponent: multiply the exponents

$$\text{Ex: } (b^7)^2 = b^{7 \cdot 2} = b^{14}$$

$$\text{Ex: } (3^4)^5 = 3^{4 \cdot 5} = 3^{20}$$

- When dividing: subtract the exponents (or cancel out)

$$\text{Ex: } \frac{c^4}{c^2} = c^{4-2} = c^2$$

$$\text{Ex: } \frac{(-3)^6}{(-3)^3} = (-3)^{6-3} = (-3)^3$$

- Negative exponents: move **ONLY** the numbers or variables that have **NEGATIVE EXPONENTS**.

$$\text{Ex: } 7x^{-4} = 7 \frac{1}{x^4}$$

$$\text{Ex: } \frac{1}{(3x)^{-2}} = (3x)^2 = \frac{(3x)^2}{1}$$

## 7-3 Practice

### *Rational Exponents*

For problems 1-6, write each expression in radical form, or write each radical in exponential form.

1.  $\sqrt{13}$

2.  $\sqrt{37}$

3.  $\sqrt{17x}$

Problems 1-3 are in radical form or exponential form? (Circle one)

---

4.  $(7ab)^{\frac{1}{2}}$

5.  $21z^{\frac{1}{2}}$

6.  $13(ab)^{\frac{1}{2}}$

Problems 4-6 are in radical form or exponential form? (Circle one)

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**Simplify.**

7.  $\left(\frac{1}{81}\right)^{\frac{1}{4}}$

8.  $\sqrt[5]{1024}$

9.  $512^{\frac{1}{3}}$

10.  $\left(\frac{32}{1024}\right)^{\frac{1}{5}}$

11.  $\sqrt[4]{1296}$

12.  $3125^{\frac{1}{5}}$

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**Solve each equation.**

13.  $3^x = 729$

14.  $4^x = 4096$

15.  $5^x = 15,625$

16.  $6^{x+3} = 7776$

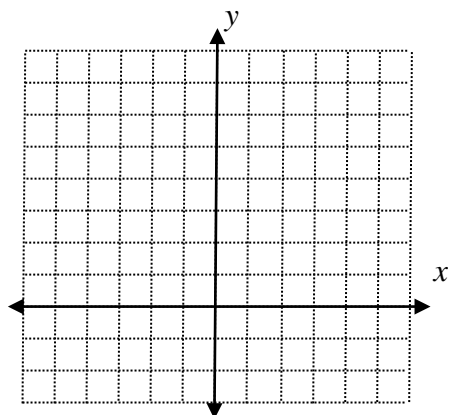
17.  $3^{x-3} = 2187$

18.  $4^{3x+4} = 16,384$

## 7.5 Exponential Functions

1) Complete the table and graph:  $y = 2^x$

$x$	-2	-1	0	—	—	—
$y$						



### Understanding:

**y-intercept:**

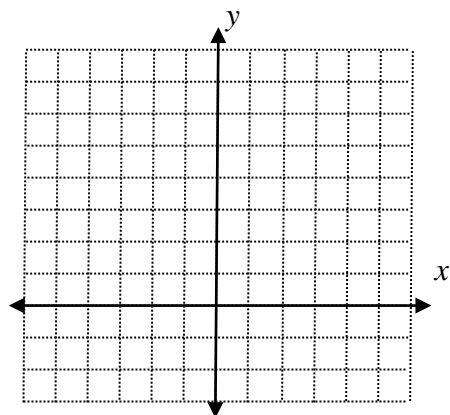
**Domain:**

**Range:**

**Is this growth or decay? Why?**

2) Complete the table and graph:  $y = \left(\frac{2}{3}\right)^x$

$x$						
$y$						



### Understanding:

**y-intercept:**

**Domain:**

**Range:**

**Is this growth or decay? Why?**



3) 1990 and 2000, the population of an endangered species decreased at a rate of 0.1% per year. The population in year is given by  $P = 1200(0.99)^t$ , where  $t = 0$  where corresponds to 1995. Find the population of the species in 1990, 1995, 2000, and the projected population in 2010.

4) Which has a bigger rate of change, an exponential function or a linear function? Support your answer below any way you can.

5) Give the data set, determine whether it represents exponential behavior. Explain why or why not.

x	0	10	20	30
y	10	25	62.5	156.25

6) The function  $v = 25000 \cdot 0.82^t$  models the depreciation in the value of a new car that originally costs \$25,000.  $V$  represents the value for the car and  $t$  represents the time in years from the time of purchase. What is the car's value after 5 years? When will the car be worth under \$100?

## 7.6

Ex. 1

- A savings certificate of \$1000 pays 6.5% annual interest compounded yearly. What is the balance when the certificate matures after 5 years?

Ex. 2

- A stock with initial cost of \$20 per share doubled its' price per share every year for 3 years.
- What is the percent of increase each year?
- What is the price per share after 3 years.

Ex.3

- From 1993 to 2007, the ratio of students per computer at a school has dropped by about 16.8% per year. If there were 103 students per computer in 1993 and 1993 is the base for comparison, what was the number of students per computer in 2007?

Ex. 4

- An experiment started with 100 bacteria. They double in number every hour.
- Write a model for the number of bacteria after  $t$  hours.
- Find the number of bacteria after 8 hours.

Ex. 5

- You bought a car for \$18000. The value of the car will be less each year because of depreciation. The car depreciates at a rate of 29% per year.
- Write an exponential decay model to represent this situation.
- Estimate the value of the car in 2 years.

- 1) You deposit \$1300 into a Chase Bank savings account that pays 4% interest. Find the balance after 3 years.
- 2) Find the balance after 3 years of an account from BMO Harris that pays 6.8% interest rate given you started the account with \$855.
- 3) Which option gives the greater ending balance?
- a) Put \$100 in an account that pays 5% interest compounded yearly for 5 years through 5/3 Bank.
  - b) Keep \$105 in your room under you mattress and add \$4 to it each year for 5 years.
- 4) A local bakery has a \$50,000 profit in 2010. Then the profit increased by 15.2% each year. Write an exponential growth model for the company's profit. Then use the model to find the profit in 2015.
- 5) In 1997, the population of Oswego was 9,572. Experts say the population increased by 8% each year. Write an exponential growth model for the town's population. Then use the model to find the projected population in 2011. The population of Oswego in 2011 was 30,856. Defend whether it was an exponential growth or not. Give specific reasons of how the numbers are similar/different.

# Unit 4 Review

For numbers 1 – 4, simplify the expression, if possible. Write your answer as a power.

1.  $2^4 \bullet 2^8$

2.  $(5^2)^7$

3.  $(9 \bullet 10)^5$

4.  $(6ab^5)^7$

For numbers 5 – 6, simplify. Then evaluate the expression when  $c = 4$  and  $d = -3$ . You will have two answers, one when you simplify, then one when you evaluate.

5.  $c^3 \bullet c$

6.  $(d^4)^2$

For numbers 7 – 8, evaluate the expression. Write your answer as a fraction in simplest form.

7.  $(3^{-3})$

8.  $(\frac{2}{5})^{-3}$

For numbers 9 – 12, rewrite the expression with positive exponents.

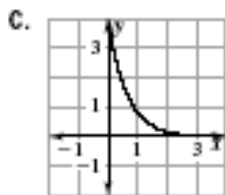
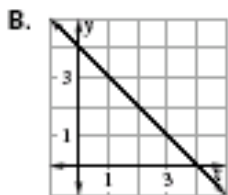
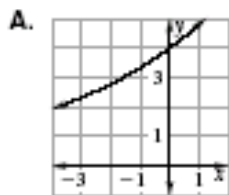
9.  $m^{-5}n^8$

10.  $\frac{3}{4g^{-8}h^{-6}}$

11.  $6^{-3}k$

12.  $\frac{9j^{-5}}{f^{-9}}$

For numbers 13 – 15, match the equation with its graph.



13.  $y = 4(1.2)^x$

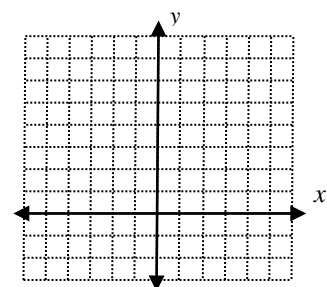
14.  $y = 4(0.2)^x$

15.  $y = 4 - x$

16. Complete the table and graph:

$y = 2\left(\frac{1}{2}\right)^x$

x					
y					

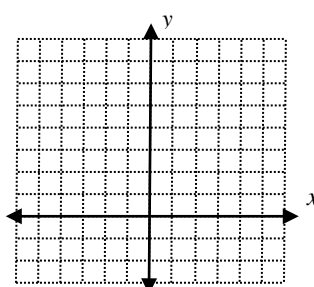


Domain:  
Range:  
y-intercept:  
growth, decay or linear?

17. Complete the table and graph:

$y = (3)^x + 2$

x					
y					



Domain:  
Range:  
y-intercept:  
growth, decay or linear?

For numbers 18 – 19, evaluate the expression. Write your answer as a fraction in simplest form.

18.  $\frac{2^6 \cdot 2^4}{2^3}$

19.  $\left(\frac{3}{10}\right)^3$

For numbers 20 – 21, simplify the expression. Write your answer with all positive exponents.

20.  $\left(\frac{5x^3y^5}{4x^2y^7}\right)^3$

21.  $\left(\frac{6x^5y^8}{7xy}\right)^{-2}$

22. You deposit \$4000 in an account that pays 6% interest compounded yearly. **Find the balance of the account after 5 years.**

23. A city had a declining population from 1992 to 1998. The population in 1992 was 200,000. Each year for 6 years, the population declined by 3%. Write an exponential model to represent this situation. Then, find the population in 1998.

For numbers 24 – 26, classify the model as exponential decay or exponential growth.

24.  $y = 10(1.35)^x$

25.  $46(1.86)^x = y$

26.  $y = 2(0.17)^x$

27. Determine whether the set of data shown below displays exponential behavior. Write yes or no. Explain why or why not.

<b>x</b>	5	0	-5	-10
<b>y</b>	3	4	35	1027

For problems 28-30, simplify. Solve 31 & 32.

28.  $64^{\frac{1}{3}}$

29.  $3x^{-\frac{5}{4}}$

30.  $\sqrt[6]{15,625}$

31.  $3^{3x+3} = 6,561$

32.  $625^x = 5$

33. Give your own example of the 7 properties of exponents:

- |    |    |    |
|----|----|----|
| 1. |    | 6. |
| 2. | 4. | 7. |
| 3. | 5. |    |

# Unit 4 Review II (Multiple Choice)

For questions 1 – 10, simplify the expression.. Assume the denominator is not equal to zero.

1.  $y^5 \cdot y^3$

A)  $y^2$

B)  $y^8$

C)  $y^{15}$

D)  $2y^8$

2.  $(b^4)^3$

A)  $b^7$

B)  $3b^4$

C)  $b^{12}$

D)  $3b^7$

3.  $\frac{a^7}{a^4}$

A)  $a^{11}$

B)  $a^{28}$

C)  $a^3$

D) 1

4.  $\frac{m^5 r^2}{m^2 r^3}$

A)  $m^7 r^5$

B)  $\frac{m^3}{r}$

C)  $m^3 r$

D)  $\frac{r}{m^3}$

5.  $(m^4)^2$

A)  $6m$

B)  $m^8$

C)  $m^6$

D)  $2m^4$

6.  $(-2xy^2)^4(2x^3y^4)^2$

A)  $4x^{24}y^{32}$

B)  $-8x^9y^6$

C)  $64x^{10}y^{16}$

D)  $-4x^{10}y^{16}$

7.  $\frac{(z^2w^{-1})^3}{(z^3w^2)^2}$

A)  $\frac{1}{w^7}$

B)  $\frac{z^{12}}{w^7}$

C)  $w$

D)  $\frac{1}{w}$

8.  $\frac{(a^{-2}b^4)^{-6}}{(a^4b^{-8})^3}$

A)  $ab^3$

B) 1

C)  $\frac{a^{24}}{b^{48}}$

D)  $\frac{b^{48}}{a^{24}}$

9.  $-\frac{12x^4y^{-3}z}{20x^8y^{-5}z^{-2}}$

A)  $-\frac{3}{5x^4y^2z^3}$

B)  $-\frac{3}{x^4y^8z}$

C)  $-\frac{3y^2z^3}{5x^4}$

D)  $-\frac{3y^8z}{5x^4}$

10.  $\frac{(-3x^3y^{-2})^3}{(9x^{-4}y^{-3})^2}$

A)  $\frac{-x^{14}y^7}{3}$

B)  $\frac{-x^2y^3}{9}$

C)  $\frac{-x}{y^{12}}$

D)  $\frac{-x^{17}}{3}$

11. Write  $6x^{\frac{1}{2}}$  in radical form.

A)  $\sqrt{6x}$

B)  $6\sqrt{x}$

C)  $6\sqrt{6x}$

D)  $x\sqrt{6}$

12. Write  $(12y)^{\frac{1}{2}}$  in radical form.

A)  $12\sqrt{y}$

B)  $\sqrt{12y}$

C)  $12\sqrt{12y}$

D)  $y\sqrt{12}$

13. Evaluate  $16^{\frac{3}{4}}$

A) 2

B) 4

C) 8

D) 32

14. Evaluate  $729^{\frac{2}{3}}$

A) 27

B) 9

C) 486

D) 81

15. Solve:  $3^{x+2} = 81$

A) 0

B) 1

C) 2

D) 3

16. Solve:  $216 = 6^{x+1}$

A) 2

B) 1

C) 4

D) 3

17. Which equation represents exponential decay?

A)  $y = 0.5x^3$

B)  $y = 0.5x^2 - x$

C)  $y = 0.5(1.07)^x$

D)  $y = 0.5(.87)^x$

18. Which equation represents exponential growth?

A)  $y = 2x^3$

B)  $y = \frac{1}{3}x^2 - x$

C)  $y = 2,000(0.82)^x$

D)  $y = 2,000(1.82)^x$

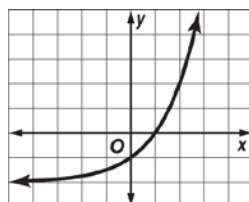
19. Which equation corresponds to the graph shown?

A)  $y = 2^x + 2$

B)  $y = 2^x - 2$

C)  $y = \left(\frac{1}{2}\right)^x - 2$

D)  $y = \left(\frac{1}{2}\right)^x + 2$



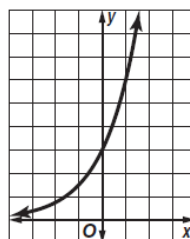
20. Which equation corresponds to the graph shown?

A)  $y = 3(2)^x$

B)  $y = 2(3)^x$

C)  $y = 3(2)^x - 1$

D)  $y = 3(2)^x + 1$



21. If  $y = 10(2.5)^t$  represents the number of bacteria in a culture at time  $t$ , how many will there be at time  $t = 6$ ?
- A) 2441                      B) 244                      C) 24                      D) none
22. A \$60,000 piece of machinery depreciates in value at a rate of 11% per year. About what will its value be in 5 years?
- A) \$47,526                      B) \$42,298                      C) \$33,504                      D) \$37,645
23. The Mendoza family just bought a house for \$180,000. If the value of the house increases at a rate of 3% per year, about how much will it be worth in 10 years?
- A) \$258,000                      B) \$241,905                      C) \$234,000                      D) \$250,000
24. If a \$5000 piece of equipment loses value at a rate of 0.5% per year, which equation represents the value after 5 years?
- A)  $y = 5000(5)^5$                       B)  $y = 5000(.995)^5$                       C)  $y = 5000(1.05)^5$                       D)  $y = 5000(.95)^5$
25. Each year, new computers are built with better technology, making older ones less valuable. If the computers loses value at a rate of 2.5% per year, how much will a \$1500 computer be worth in ten years?
- A) \$1165                      B) \$1920                      C) \$84.47                      D) \$13970
26. Which statement best describes the equation  $y = A(3.2)^x$ , where  $A$  represents the initial value and  $x$  represents time in years?
- A)  $y$  represents exponential decay of 2.2%                      B)  $y$  represents growth of 22%  
 B)  $y$  represents exponential growth of 220%                      C)  $y$  represents growth of 3.2%
27. Which expression is equivalent to  $(\sqrt[6]{x})^5$  in exponential form?
- A)  $6^{\frac{5}{x}}$                       B)  $5^{\frac{x}{6}}$                       C)  $x^{\frac{5}{6}}$                       D)  $x^{\frac{6}{5}}$

28. In the year 2000, a scientist determined there where 1500 of a certain type of deer in a forest preserve. The table shows the deer increased exponentially at a rate of 5% each year.

Number of Years since 2000 ( $t$ )	Deer Population ( $N$ )
0	1500
1	1575
2	1654

Based on on this information, which equation can be used to predict the deer population 3 years after 2000?

- A)  $N = 1500(1.5)^3$                       B)  $N = 1500(.95)^3$                       C)  $N = 1500(1.05)^3$                       D)  $N = 1500(.5)^3$



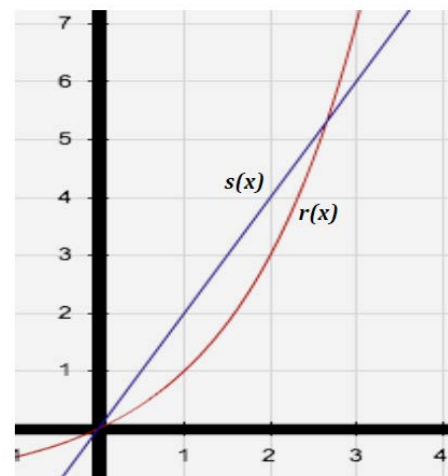
29. In the graph,  $s(x)$  is a linear function and  $r(x)$  is exponential. Which statement best explains the behavior of the graphs of the functions as  $x$  increases?

A)  $r(x)$  eventually exceeds  $s(x)$  because rate of change of  $s(x)$  increases, where as the rate of change of  $r(x)$  is constant.

B)  $r(x)$  eventually exceeds  $s(x)$  because the rate of change of  $r(x)$  increased as  $x$  increases, whereas the rate of change of  $s(x)$  is constant.

C)  $s(x)$  eventually exceeds  $r(x)$  because rate of change of  $r(x)$  increases, Where as the rate of change of  $s(x)$  is constant.

D)  $s(x)$  eventually exceeds  $r(x)$  because rate of change of  $s(x)$  increases, Where as the rate of change of  $r(x)$  is constant.



30. Samuel won a contest where he wins a yearly prize for his lifetime. Samuel can choose to be paid \$5000 per year (option 1 in the table below) or his payments can be tripled each year, with the first year the payment starting at \$100. (Assume Samuel is 15 and will live to be 100 years old)

Year	Option 1	Option 2
1	\$5000	\$100
2	\$5000	\$300
3	\$5000	\$900

Which prize option should Samuel choose in order to earn the most money over his lifetime?

A) Option 1 because the total payment is increasing exponentially

B) Option 1 because the total payment is increasing linearly

C) Option 2 because the total payment is increasing exponentially

D) Option 2 because the total payment is increasing linearly

31. The graph and table for  $y = \left(\frac{1}{4}\right)^x$  is shown. What is the domain and range?

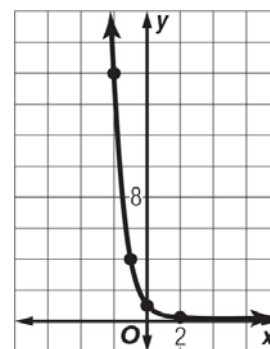
A) D:  $x > 0$  , R:  $y > 0$

B) D:  $y > 0$  , R: All Real Numbers

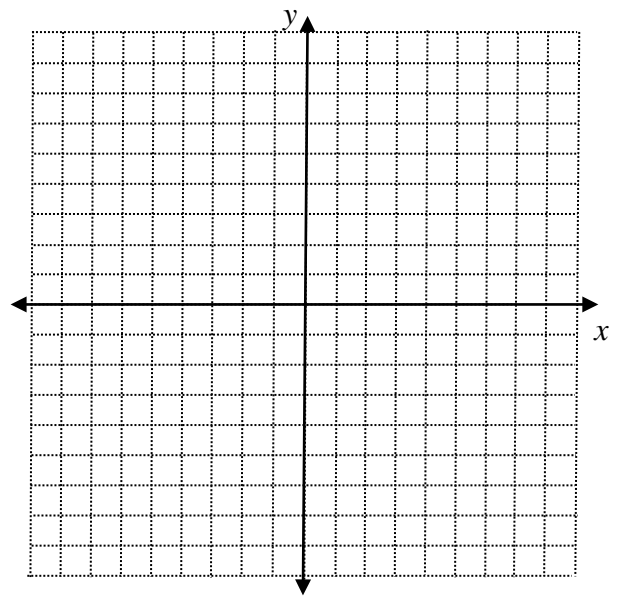
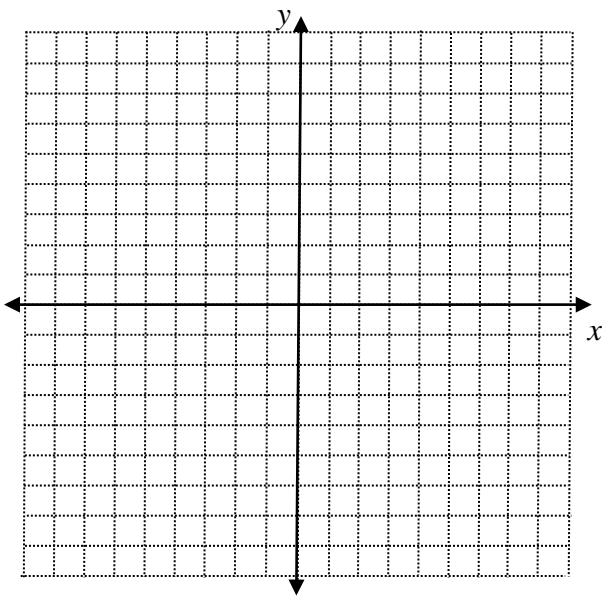
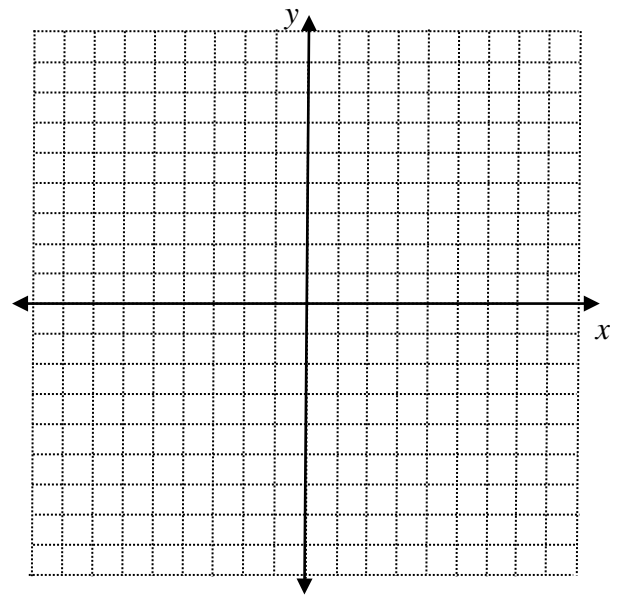
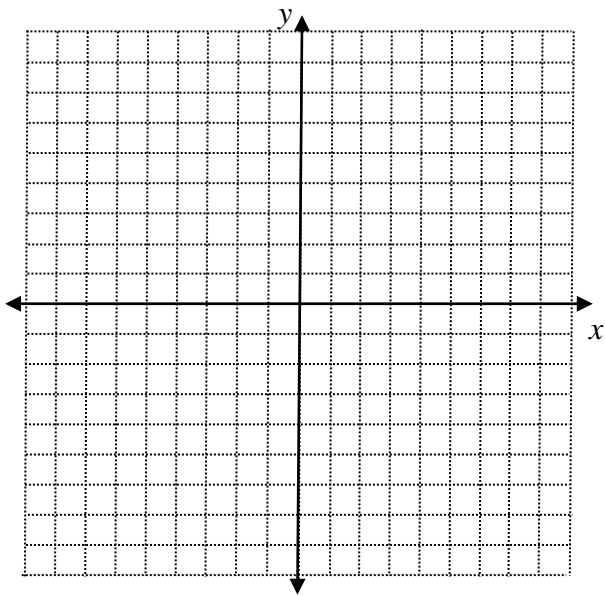
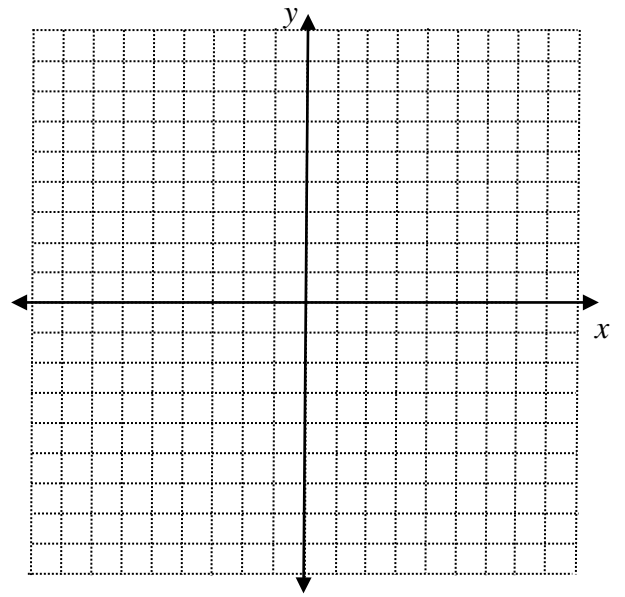
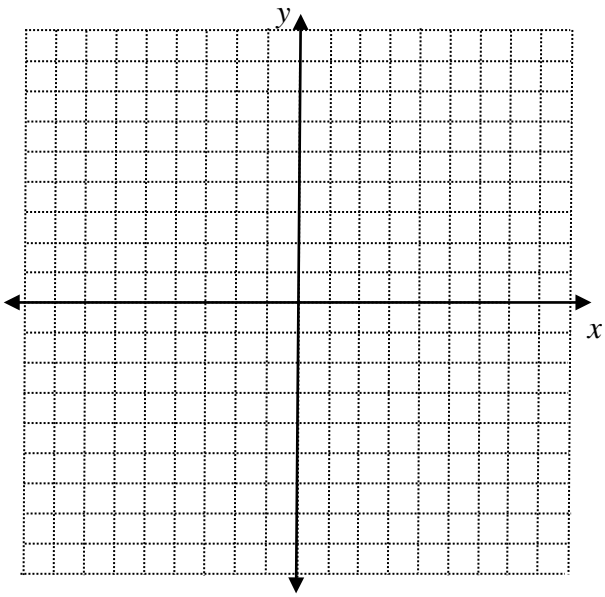
C) D: All Real Numbers, R: All Real Numbers

D) D: All Real Numbers, R:  $y > 0$

$x$	$y$
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



# Extra Graphs



Add on problems or addition examples:

Add on problems or addition examples: