Pre-Calculus
REVIEW: Chapter 10 – Sequences & Series

Name Key

1. Which term of the arithmetic sequence 3, 8, 13,18,23, …, 113 is 113?

A) 20th B) 21st C) 22nd D) 23rd

$a_1 = 3$ $d = 5$ $a_n = 3+(n-1)(5)$ $a_n = 5n-2$ $113 = 5n-2$ $n = 23$

2. Find the 27th term in the arithmetic sequence –8, 1, 10, …

A) 174 B) 226 C) 235 D) 242

$a_1 = -8$ $d = 9$ $a_n = -8+(n-1)(9)$ $a_n = 9n-17$ $a_{27} = 9(27)-17 = 226$

3. In an arithmetic sequence, what is d if $a_1 = 14$ and $a_{24} = 50.8$?

A) 1.6 B) 2.1 C) 2.6 D) 3.6

$50.8 = 14+(24-1)d$ $d = 1.6$

4. Find S_{73} in the arithmetic series –0.2 + 0.3 + 0.8 + … .

$a_1 = -.2$ $d = .5$ $a_{73} = -.2+(73-1)(.5)$

A) 3186 B) 1306.7 C) 1299.4 D) 1317.65

$S_{73} = \frac{73}{2}(-.2+35.8)$

5. An employee's salary increases by the same amount each year. If he earned \$77,900 for the seventh year and \$97,500 for the fifteenth year, how much was his pay for the second year?

A) \$61,100 B) \$63,200 C) \$63,900 D) \$65,650

$a_7 = 77,900$ $a_{15} = 97,500$ $a_2 = ?$

$77900 = a_1+(7-1)d$ $97,500 = a_1+(15-1)d$

$77900 = a_1+6d$ $97500 = a_1+14d$

$-77900 = -a_1-6d$

$19600 = 8d$ $d = 2450$ $a_1 = 63200$

6. Write $\sum_{n=2}^{4} 5(\frac{2}{3})^n$ in expanded form and then find the sum.

A) $5\left(\frac{4}{9}\right)+\left(\frac{4}{9}\right)+\left(\frac{4}{9}\right); \frac{28}{9}$

B) $\left(\frac{10}{3}\right)^2+\left(\frac{10}{3}\right)^3+\left(\frac{10}{3}\right)^4; \frac{15,700}{81}$

C) $5\left(\frac{2}{3}\right)^1+5\left(\frac{2}{3}\right)^2+5\left(\frac{2}{3}\right)^3; \frac{190}{27}$

D) $\left(\frac{20}{9}\right)+\left(\frac{40}{27}\right)+\left(\frac{80}{81}\right); \frac{380}{81}$

$5\left(\frac{2}{3}\right)^2+5\left(\frac{2}{3}\right)^3+5\left(\frac{2}{3}\right)^4$

$5\left(\frac{4}{9}\right)+5\left(\frac{8}{27}\right)+5\left(\frac{16}{81}\right)$

7. Express the series $0.7 + 0.007 + 0.00007 + \ldots$ using sigma notation. Geo!

$a_1 = .7$

$r = \frac{1}{100}$

$\sum_{n=1}^{\infty} .7\left(\frac{1}{100}\right)^{n-1}$

8. Given $a_3 = -16$ and $a_{27} = -304$ in an arithmetic sequence, find a_{64}.

$-16 = a_1 + 2d$

$-304 = a_1 + 26d$

$+\ 16 = -a_1 - 2d$

$d = -12$

$a_1 = 8$

$a_{64} = 8 + (64-1)(-12)$

$= -748$

9. Find the sum of $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$.

$a_1 = 3$

$r = \frac{-1}{3}$

$S_n = \frac{3}{1-\left(\frac{-1}{3}\right)} = \frac{9}{4}$

10. Find the 40th term of the arithmetic sequence $7, \frac{22}{5}, \frac{9}{5}, -\frac{4}{5}, \ldots$.

$a_1 = 7$

$d = -\frac{13}{5}$

$a_{40} = 7 + (40-1)\left(\frac{-13}{5}\right)$

$-\frac{477}{5}$

11. If a_1 is 6 and $a_{13} = -42$, find the common difference d.

$-42 = 6 + (13-1)d$

$d = -4$

12. Write the arithmetic series using summation notation and then find the sum of the first 30 terms of $10 + 6.8 + 3.6 + \ldots$

$\sum_{n=1}^{30} -3.2n + 13.2$

$S_{30} = \frac{30}{2}(10 + 82.8)$

-1092

$a_1 = 10$

$d = -3.2$

$a_n = 10 + (n-1)(-3.2)$

13. An employee agreed to a salary plan where his annual salary increases by the same amount each year. If he earned \$51,100 for the fifth year and \$64,900 for the eleventh year, how much will he have earned in total after 25 years at his job?

$a_5 = 51{,}100$ $a_{11} = 64{,}900$ $a_{25} =$

$51{,}100 = a_1 + 4d$

$64{,}900 = a_1 + 10d$

$13800 = 6d$

$d = 2300$

$a_1 = 41900$

$a_{25} = 41900 + (25-1)(2300)$

$a_{25} = \$97100$

$S_{25} = \frac{25}{2}(41800 + 97100)$

$\$1{,}737{,}500$

14. Find the sum of the first eight terms in the geometric series $64 - 32 + 16 - 8 \ldots$.

$a_1 = 64$

$r = -\frac{1}{2}$

$S_8 = 64\left(\frac{1-\left(-\frac{1}{2}\right)^8}{1-\left(-\frac{1}{2}\right)}\right)$

$= \frac{85}{2}$ or 42.5

15. Write $\sum_{n=2}^{7} 81(-\frac{1}{3})^{n-2}$ in expanded form. Then find the sum.

Geo

$81\left(-\frac{1}{3}\right)^{0} + 81\left(-\frac{1}{3}\right)^{1} + 81\left(-\frac{1}{3}\right)^{2} + 81\left(-\frac{1}{3}\right)^{3} + 81\left(-\frac{1}{3}\right)^{4} + 81\left(-\frac{1}{3}\right)^{5} = \frac{182}{3}$

16. Find n for $8+5+2+-1.....$, where $a_n = -40$.

$a_1 = 8$

$d = -3$

$-40 = 8 + (n-1)(-3)$

$-51 = -3n$

$n = 17$

17. In a geometric series, $t_3 = 9$ and $t_6 = 30.375$, find S_{17}.

$9 = a_1 \cdot r^2$ $\quad 30.375 = a_1 \cdot r^5$

$30.375 = \left(\frac{9}{r^2}\right) r^5$

$3.375 = r^3$

$1.5 = r$

$4 = a_1$

$S_{17} = 4\left(\frac{1-(1.5)^{17}}{1-(1.5)}\right)$

≈ 787409

18. Find n for $5 - 15 + 45 - 135$ ..... If $S_n = 2735$

$a_1 = 5$

$r = -3$

Geo

$2735 = 5\left(\frac{1-(-3)^n}{1-(-3)}\right)$

$2188 = 1-(-3)^n$

$-2187 = (-3)^n$

$\log_3 2187 = n$

$n = 7$

19. Use the series: $2 + 3 + 4 + 5 + 6 + ...$ find n when $S_n = 5150$.

Arith

$a_1 = 2$

$d = 1$

$a_n = 2 + (n-1)(1)$

$a_n = n + 1$

$5150 = \frac{n}{2}(2 + a_n)$

$10300 = n(2 + n + 1)$

$0 = n^2 + 3n - 10308$

100

-103

Quad form / Bev Quad / Graph

20. Determine if each of the following series converges or diverges.

a. $18, 28, 38, 48, ...$

$d = 10$

Diverge

b. $-3, \frac{12}{5}, -\frac{48}{25}, \frac{192}{125}...$

$r = \frac{-4}{5}$

C

c. $81, 27, 9, 3, ...$

$r = \frac{1}{3}$

C

d. $\sum_{n=1}^{\infty} \frac{16}{9}(\frac{3}{2})^{n-1}$

r

D

Answer Key

1) D 2) B 3) A 4) C 5) D 6) D 7) $\sum_{n=1}^{\infty}(0.7)(.01)^{n-1}$ 8) -748 9) $\frac{9}{4}$

10) $\frac{-472}{5}$ 11) -4 12) $\sum_{n=1}^{30}-3.2n+13.2$;–1092 13) \$1,737,500 14) 42.5

15) $81-27+9-3+1-\frac{1}{3}$; Sum $=\frac{182}{3}$ 16) 17 17) 7874.09 18) 7 19) 100

20 a.diverges b. converges c. converges d. diverges

FORMULAS:

$a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}(a_1 + a_n)$

$a_n = a_1(r)^{n-1}$

$S_n = a_1(\frac{1-r^n}{1-r})$

$S = \frac{a_1}{1-r}$