Pre-Calculus Unit G (Chapter 5) Review sheet Name Key

1. Find the exact value of cos 15°. $\cos(45°-30°) = \cos45°\cos30° + \sin45°\sin30°$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

F $\frac{-\sqrt{2}-\sqrt{6}}{4}$ **G** $\frac{\sqrt{6}-\sqrt{2}}{4}$ **H** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **J** $\frac{\sqrt{2}-\sqrt{6}}{4}$

2. Solve $\sin x - \sin^2 x = 0$ on the interval $[0, 2\pi)$.

$\sin x(1-\sin x) = 0$

$\sin x = 0$ | $\sin x = 1$

$0, \pi$ | $\frac{\pi}{2}$

A $0, \pi, \frac{\pi}{2}$ **B** $\frac{\pi}{2}$ **C** $\frac{\pi}{2}, \frac{3\pi}{2}$ **D** $\pi, \frac{\pi}{4}$

3. Find the exact value of $\cos\frac{\pi}{8}$. $\cos\frac{\frac{\pi}{4}}{2} = \pm\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{2+\sqrt{2}}{4}}$

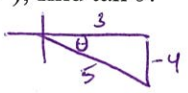
$= +\frac{\sqrt{2+\sqrt{2}}}{2}$ (Q1)

F $-\frac{\sqrt{2-\sqrt{2}}}{2}$ **G** $-\frac{\sqrt{2+\sqrt{2}}}{2}$ **H** $\frac{\sqrt{2-\sqrt{2}}}{2}$ **J** $\frac{\sqrt{2+\sqrt{2}}}{2}$

4. Simplify $\frac{\sec\theta\tan\theta}{\sin\theta}$. $= \frac{\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)}{\sin\theta} = \frac{\frac{\sin\theta}{\cos^2\theta}}{\sin\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$

A $\sec^2\theta$ **B** $\cot\theta$ **C** $\tan^2\theta$ **D** $\cos^2\theta$

5. If $\csc\theta = -\frac{5}{4}$ on the interval $(270°, 360°)$, find $\tan\theta$.

$\sin\theta = \frac{-4}{5}$ (Q4) 

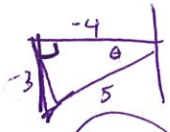
$\tan\theta = \frac{-4}{3}$

F $-\frac{4}{3}$ **G** $\frac{3}{4}$ **H** $\frac{4}{3}$ **J** $-\frac{4}{5}$

6. Simplify $\frac{\cos x}{\sec x - 1}\cdot\frac{(\sec x+1)}{(\sec x+1)} + \frac{\cos x}{\sec x + 1}\cdot\frac{(\sec x-1)}{(\sec x-1)} = \frac{\cos x\sec x + \cos x + \cos x\sec x - \cos x}{\sec^2 x - 1} = \frac{2\cos x\sec x}{\tan^2 x} = \frac{2}{\tan^2 x} =$

F $2\tan^2 x$ **G** $2\cos x$ **H** $2\cos^2 x - 1$ **J** $2\cot^2 x$

7. If $\csc\theta = -\frac{5}{3}$ on the interval $\left(\pi, \frac{3\pi}{2}\right)$, find the exact value of $\tan 2\theta$.

$\sin\theta = \frac{-3}{5}$ (Q3) 

$\tan 2\theta = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$

A $\frac{24}{25}$ **B** $-\frac{7}{25}$ **C** $\frac{24}{7}$ **D** $\frac{7}{25}$

8. Solve $2\sin x - \sqrt{3} = 0$ on the interval $[0, 2\pi)$.

$+\sqrt{3}\ +\sqrt{3}$

$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi$ $\frac{\pi}{3}, \frac{2\pi}{3}$

A $\frac{\pi}{6}, \frac{5\pi}{6}$ **B** $\frac{\pi}{3}, \frac{2\pi}{3}$ **C** $\frac{\pi}{3}, \frac{5\pi}{3}$ **D** $\frac{7\pi}{6}, \frac{11\pi}{6}$

9. Find the exact value of $\cos 67.5°$.

$\cos\frac{135^\circ}{2} = \pm\sqrt{\frac{1+\cos 135^\circ}{2}} = \sqrt{\frac{1+\frac{-\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

F $-\frac{\sqrt{2-\sqrt{2}}}{2}$ **G** $-\frac{\sqrt{2+\sqrt{2}}}{2}$ **H** $\frac{\sqrt{2-\sqrt{2}}}{2}$ **J** $\frac{\sqrt{2+\sqrt{2}}}{2}$

10. Verify: $\cos\theta\tan^2\theta + \cos\theta = \sec\theta$

$\cos\theta(\tan^2\theta + 1) =$

$\cos\theta(\sec^2\theta) =$

$\cos\theta\left[\frac{1}{\cos\theta}\right]$ $\sec\theta = \sec\theta$ ✓

11. Simplify $\csc\theta - \cot\theta\cos\theta$.

$\frac{1}{\sin x} - \frac{\cos x}{\sin x}(\cos x)$

$\frac{1-\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x}$

$\sin x$

12. Find the exact value of $\cos 255°$. $\cos(225° + 30°)$

$\cos 225^\circ\cos 30^\circ - \sin 225^\circ\sin 30^\circ$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

13. Find the exact value of $\dfrac{\tan 25^\circ + \tan 35^\circ}{1 - \tan 25^\circ\tan 35^\circ}$

$\tan(25° + 35°)$

$\tan(60°)$ $\sqrt{3}$

14. Solve on the interval $[0, 2\pi)$: $\cos 2\theta + 2\cos^2\theta = 0$

$(2\cos^2\theta - 1) + 2\cos^2\theta = 0$

$4\cos^2\theta - 1 = 0$

$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{4}}$

$\cos\theta = \pm\frac{1}{2}$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

15. Solve on the interval $[0, 2\pi)$: $\cos\theta \sin 2\theta = 0$

$\cos\theta[2\sin\theta\cos\theta] = 0$

$2\cos^2\theta\sin\theta = 0$

$2\sin\theta[1-\sin^2\theta] = 0$

$2\sin\theta = 0$ → $0, \pi$

$1-\sin^2\theta = 0$

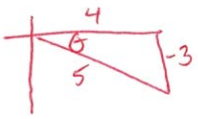
$\sin\theta = \pm 1$ → $\frac{\pi}{2}, \frac{3\pi}{2}$

16. Verify: $\frac{1-\sin^2\theta}{1-\cos^2\theta} = \cot^2\theta$.

$\frac{\cos^2\theta}{\sin^2\theta}$

$\cot^2\theta = \cot^2\theta$ ✓

17. If $\cos\theta = \frac{4}{5}$ on the interval $\left(\frac{3\pi}{2}, 2\pi\right)$, find the exact value of $\tan 2\theta$.



Q4

$\frac{2\left(\frac{-3}{4}\right)}{1-\left(\frac{-3}{4}\right)^2}$

$\frac{\frac{-3}{2}}{\frac{7}{16}}$ → $\frac{-24}{7}$

18. Solve $\tan x - \sqrt{3} = 0$ on the interval $[0, 2\pi)$.

$\tan x = \sqrt{3}$

$\frac{\pi}{3}, \frac{4\pi}{3}$

19. Find the exact value of $\cos\frac{7\pi}{12}$.

$\cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

For 20-22, Find all solutions on the interval $[0, 2\pi)$.

20. $\sin^2 x - 2\sin x = 3$

$-3 \quad -3$

$(\sin x - 3)(\sin x + 1) = 0$

$\sin x = 3$ → no sol'n

$\sin x = -1$ → $\frac{3\pi}{2}$

21. $\sin^2 x - \sin x + 1 = \cos^2 x$ $[1-\sin^2 x]$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0 \quad | \quad \sin x = \frac{1}{2}$

$0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

22. $4\cos^2 x - 1 = 0$ see #14

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

For 22-25, Find the exact value of each trigonometric expression.

23. $\cos\frac{5\pi}{12}$ $\cos\left(\frac{8\pi}{12} - \frac{3\pi}{12}\right)$

$\cos\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$

$\cos\frac{2\pi}{3}\cos\frac{\pi}{4} + \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{-\sqrt{2}+\sqrt{6}}{4}$

24. $\sin(195°)$ $\sin(225° - 30°)$

$\sin 225°\cos 30° - \cos 225°\sin 30°$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\frac{-\sqrt{6}+\sqrt{2}}{4}$

25. $\tan 345°$

$\tan(300° + 45°)$

$\frac{\tan 300° + \tan 45°}{1 - \tan 300°\tan 45°}$

$\frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$ $\frac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$\frac{-\sqrt{3} + \sqrt{9} + 1 - \sqrt{3}}{1 - \sqrt{9}}$

$\frac{-2\sqrt{3} + 4}{-2}$

$+\sqrt{3} - 2$

26. $\tan\left(\frac{23\pi}{12}\right)$ $\tan\left(\frac{8\pi}{12} + \frac{15\pi}{12}\right)$ or $\tan\left(\frac{2\pi}{3} + \frac{5\pi}{4}\right)$

$\frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$

see #25

$\sqrt{3} - 2$